Spin Glasses in all Dimensions

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Overview:

• Bond-diluted Edwards-Anderson (EA) Spin Glasses
  ➔ Defect-Energy Scaling  \( \sigma(\Delta E) \sim L^y \)
  ➔ Reduction of large diluted Lattices at \( T=0 \)
  ➔ “Stiffness” Exponent \( y \) for \( d=3,...,7 \)
  ➔ Lower critical dimension \( d_l=5/2 \)
  ➔ Corrections-to-Scaling Exponent \( \omega \)

• Sherrington-Kirkpatrick (SK) Mean-Field Spin Glass
  ➔ Ground State Energies
  ➔ Scaling Corrections + Energy Fluctuations
  ➔ Comparison with EA at \( d \to \infty \)
Lattice Spin Glasses (at $T=0$):

**Defect-Energy:**

![Diagram showing a grid with energy $E_0$ at specific coordinates]
Lattice Spin Glasses (at $T=0$):

**Defect-Energy:**

Measure Defect Energy $\Delta E = E_0 - E'_0$
Lattice Spin Glasses (at $T=0$):

**Defect-Energy:** Measure Defect Energy $\Delta E = E_0 - E'_0$  \[ \Rightarrow \sigma(\Delta E) \sim L^y \]

- Low Energy Excitations (like “small oscillations”)

![Graph showing energy levels and defect energy calculation](image)
Lattice Spin Glasses (at $T=0$):

Defect-Energy:

Low Energy Excitations of bond-diluted Lattices
Lattice Spin Glasses (at $T=0$):

Defect-Energy: Collect $\sigma(\Delta E) \sim L^y$

Before: 100 Spins

Low Energy Excitations of bond-diluted Lattices
Lattice Spin Glasses (at $T=0$):

**Defect-Energy:**

Collect $\sigma(\Delta E) \sim L^y$

Before: 100 Spins

After: 5 Spins

“Reduction” Algorithm (exact!) & Optimization Heuristic ($\tau$-EO)

Low Energy Excitations of bond-diluted Lattices
Lattice Spin Glasses (at $T=0$):

**Defect-Energy:** Measure "Stiffness": $\sigma(\Delta E) \sim L^y$

*How bond-diluted Lattices?*

*Why bond-diluted Lattices?*

- Simpler Problem
- Larger Sizes $L$
- Better Scaling

$\sigma$ (Stress), $\Delta E$ (Energy), $L$ (Length), $T_g$ (Glass Transition Temperature), $T=0$ (Zero Temperature), $p_c$ (Critical Bond Density), $\rho$ (Bond Density), $y>0$ (Exponent for Scaling), PM (Paramagnetic), SG (Spin-Glass)
Defect-Energy of diluted Lattices:

- $\pm J$-Glasses on Lattices of size $L$ and density $p$.
- Defect-Energy $\sigma(\Delta E)$ with Reduction & Heuristic ($\tau$-EO).
Stiffness Exponent $y$ for Lattice Glasses:

\[ \text{“Stiffness”: } \sigma(\Delta E) \sim L^y \]

\[ y_3 = 0.24(1) \]
**Stiffness Exponent $y$ for Lattice Glasses:**

“Stiffness”: $\sigma(\Delta E) \sim L^y$

- $d=3$: $y_3 = 0.24(1)$
- $d=4$: $y_4 = 0.61(1)$
Stiffness Exponent $y$ for Lattice Glasses:

\[ d=3 \]
\[ y^3 = 0.24(1) \]

\[ d=4 \]
\[ y^4 = 0.61(1) \]
\[ y^5 = 0.88(5) \]

\[ d=5 \]

\[ \text{“Stiffness”: } \sigma(\Delta E) \sim L^y \]
Stiffness Exponent $y$ for Lattice Glasses:

“Stiffness”: $\sigma(\Delta E) \sim L^y$

$\begin{align*}
\text{d}=3 \\
y_3 &= 0.24(1) \\
y_4 &= 0.61(1) \\
y_5 &= 0.88(5) \\
y_6 &= 1.1(1)
\end{align*}$

$\begin{align*}
\text{d}=4 \\
\text{d}=5 \\
\text{d}=6
\end{align*}$
Stiffness Exponent $\gamma$ for Lattice Glasses:

```
\text{"Stiffness"}: \sigma(\Delta E) \sim L^\gamma
```

For $d=3$:
- $y_3 = 0.24(1)$

For $d=4$:
- $y_4 = 0.61(1)$
- $y_5 = 0.88(5)$

For $d=5$:
- $y_6 = 1.1(1)$

For $d=6$:
- $y_7 = 1.24(5)$

For $d=7$:
- $y_7 = 1.24(5)$
Comparing with Theory:

“Stiffness”: $\sigma(\Delta E) \sim L^y$

$d_{\text{lower}} = 5/2$

$y_{\text{RSB}} = d(1-\rho)$
Other Evidence for $d_l=5/2$:

- **From Theory:** (Franz, Parisi & Virasoro, J. Phys. I 4, 1657, '94)
  Effective Mean Field calculation near $T_g$, where Replica Symmetry Breaking (RSB) disappears (i.e. $T_g \rightarrow 0$) for $d_l=5/2$.

- **From Numerics:**

  Know:
  
  $T_g \approx \sqrt{2d}$ \hspace{1cm} (d \rightarrow \infty)
  
  $T_g \approx \sqrt{d-d_l}$ \hspace{1cm} (d \rightarrow d_l)

  Data from:
  
  - MC (Ballesteros et al) for d=3,4
  - High-T Series (Klein et al) for d\geq5
Corrections to Scaling in Spin Glasses:

Ground State Energy: \( E(L) \sim e_0 L^d + A L^y \) \((L \rightarrow \infty)\)

**Diagram**

- \( d=5 \)
- \( p=0.13 \)
- \( L=7 \ldots 15 \)
Comprehensive View on Spin Glasses:

A Set of Exponents:

1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$
   - In EA: $\rho = \frac{1}{2}$ (Wehr&Aizenman $\rightarrow$ exact!)

2) Distribution $P(\Delta E_0)$, width $\sigma(\Delta E_0) \sim N^{y/d} = L^y$
   - In EA: $y \approx 0.24,...,1.2$ for $d=3,...,7$

3) Corrections-to-Scaling: $e_0(N)-e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$
   - In EA: $\omega/d = 1 - y/d$
\( \tau \)-EO for Sherrington-Kirkpatrick (SK):

- Mean-Field \( (d \to \infty) \) Spin Glasses:
Sherrington-Kirkpatrick (SK) at T=0:

- PDF for $E_0$ from *exact* Enumeration:
τ-EO for Sherrington-Kirkpatrick (SK):

- Mean-Field \((d \to \infty)\) Spin Glasses:

Fluctuation Exponent \(\rho = 3/4\)
Comparing with Theory:

“Stiffness”: \( \sigma(\Delta E) \sim L^y \)
Comprehensive View on Spin Glasses:

1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$
   - In EA: $\rho = \frac{1}{2}$ (Gaussian)
   - In SK: $\rho \approx \frac{3}{4}$ (Highly Skewed)

2) Distribution $P(\Delta E_0)$, width $\sigma(\Delta E_0) \sim N^{y/d} = L^y$
   - In EA: $y \approx 0.24, \ldots, 1.2$ for $d=3, \ldots, 7$
   - In SK: $y/d = 1 - \rho \rightarrow 1/4$, too high for EA at $d \rightarrow \infty$

3) Corrections-to-Scaling: $e_0(N) - e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$
   - In EA: $\omega/d = 1 - y/d$
   - In SK: $\omega/d \approx 2/3 \neq 1 - y/d$
\textbf{\tau-EO for Bethe Lattices:}

EO for 3-connected Bethe Lattice Glass w/ Replica Sym. Breaking:

\begin{itemize}
  \item EO
  \item fit
  \item 1RSB
  \item RS
\end{itemize}

$\varepsilon_3$ vs $1/n^{2/3}$

- EO data,
  - [PRB67(03)060403R]

- Replica Theory:
  - $\iff 1$RSB,
  - $\iff$ no RSB,
    - [J.Stat.Phys.111(03)1]

Scaling corrections
\textbf{\texttau-EO for Bethe Lattices:}

GS variance for $z$-connected Bethe Lattice Glass:

$\sigma/N^{1/2}$ vs $N/z^{2.3}$

- $\rho \approx 3/4$
- $z \sim N$ limit (SK only!)
- $z$ finite (Bethe L)
- $\rho = 1/2$
Conclusions:

- **Bond-Diluted Lattices:**
  - Reducing dilute Lattices $\Rightarrow$ reach larger $L$
  - Finite-Size Scaling $\Rightarrow$ extended Scaling Regime
  - **Accurate Scaling Exponents** (Stiffness $y$, CtS $\omega$) ... 
  - ...even in high dimensions
  - Allows **Comparison with MFT**
  - Allows Prediction of **lower critical dimension** $d_t=5/2$.

**Outlook:**

- **Reduction Algorithm:**
  - Determining $T_g \sim (p-p_c)^\phi$ $\Rightarrow$ Experimentally Testable!
  - Determining fractal exponent of droplets.