Spin Glasses in all Dimensions

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(Pre-)Prints: *www.physics.emory.edu/faculty/boettcher*

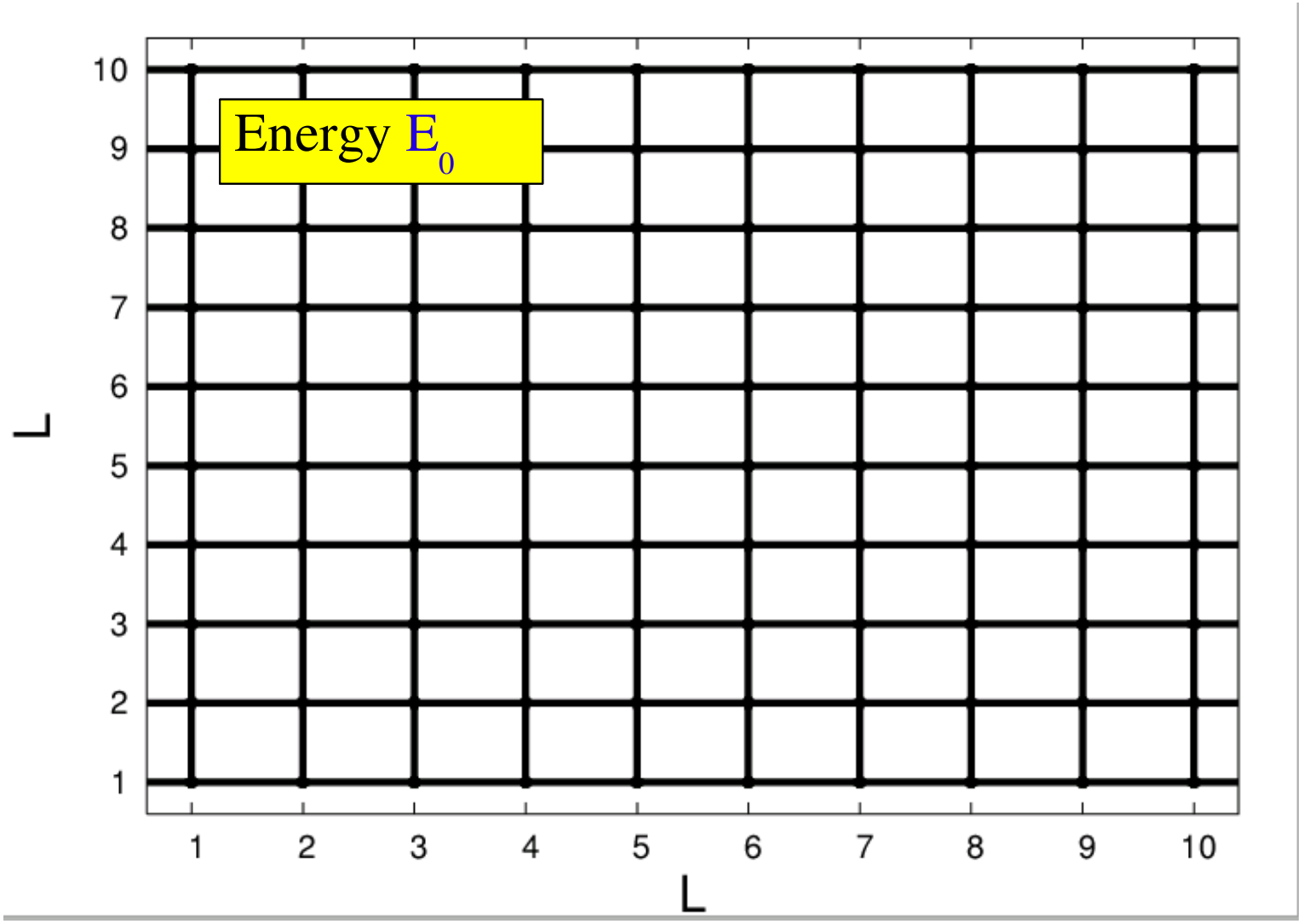
Overview:

- Bond-diluted Edwards-Anderson (**EA**) Spin Glasses
  - Defect-Energy Scaling $\sigma(\Delta E) \sim L^y$
  - Reduction of large diluted Lattices at $T=0$
  - "Stiffness" Exponent y for d=3,...,7
  - Lower critical dimension $d_l=5/2$
  - Corrections-to-Scaling Exponent ω

- Sherrington-Kirkpatrick (**SK**) Mean-Field Spin Glass
  - Ground State Energies
  - *Scaling Corrections + Energy Fluctuations*
  - Comparison with **EA** at $d \rightarrow \infty$

Lattice Spin Glasses (at $T=0$):

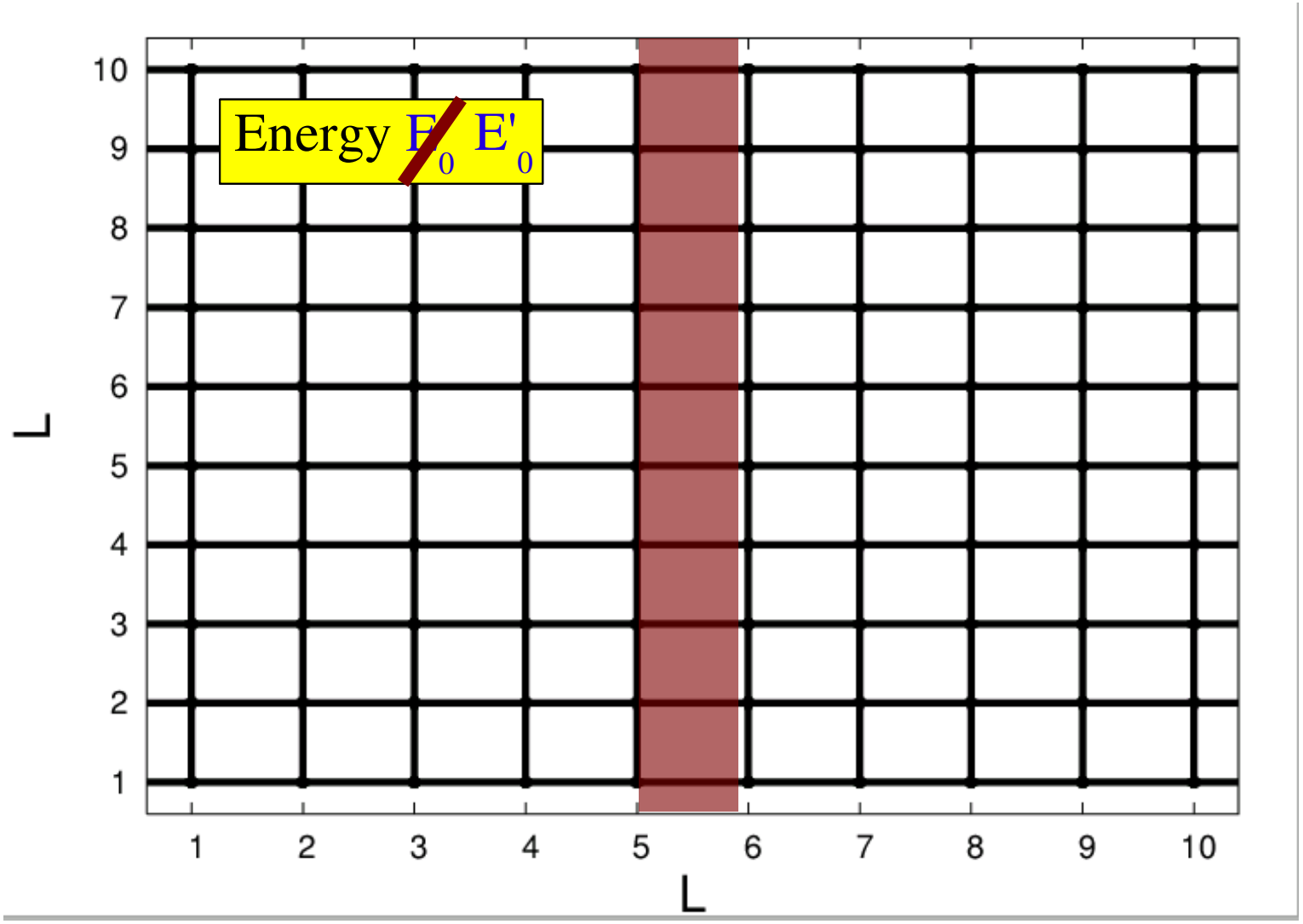
Defect-Energy:



Lattice Spin Glasses (at $T=0$):

Defect-Energy:

Measure Defect Energy $\Delta E = E_0 - E'_0$

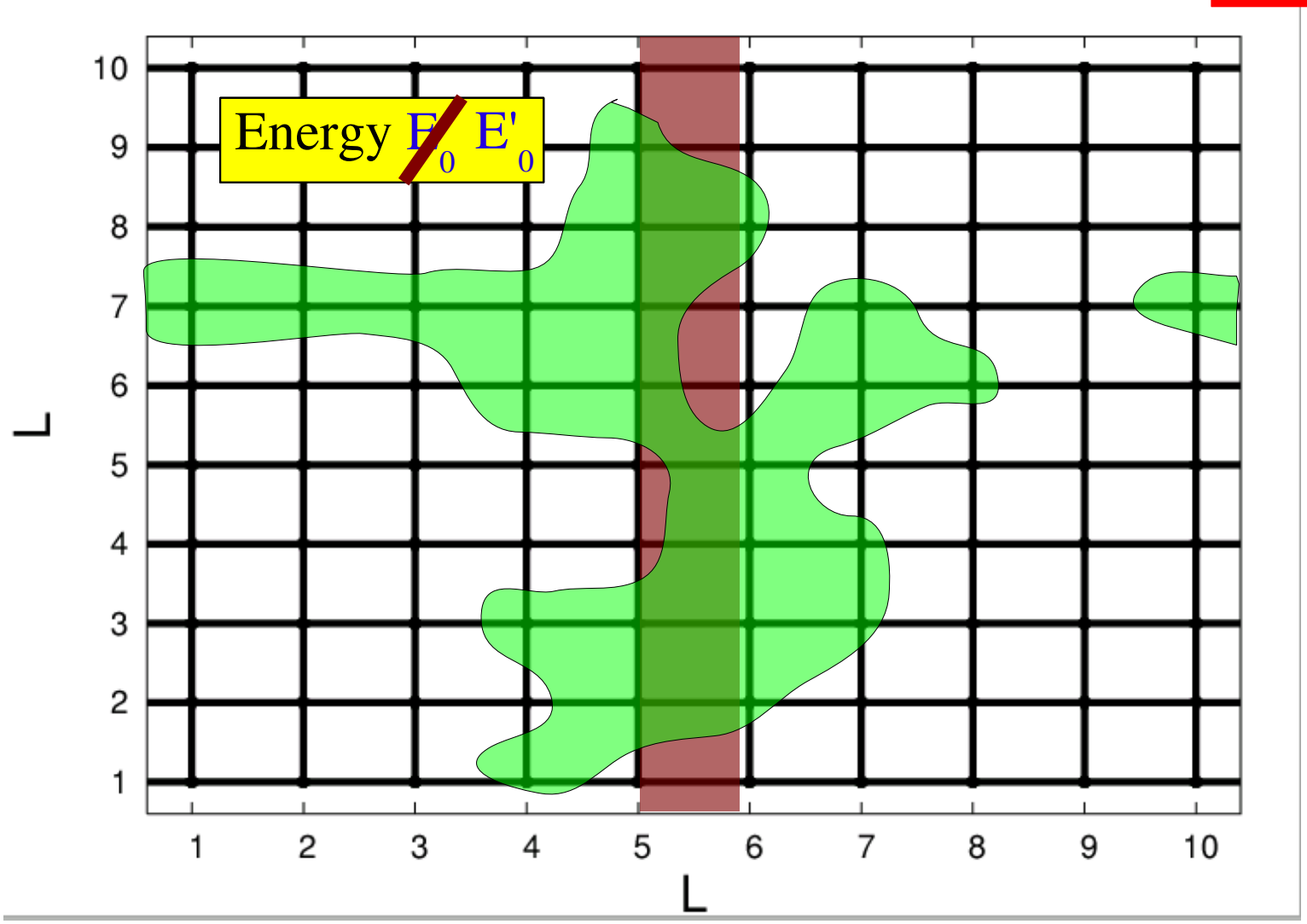


Lattice Spin Glasses (at $T=0$):

Defect-Energy:

Measure Defect Energy $\Delta E = E_0 - E'_0$

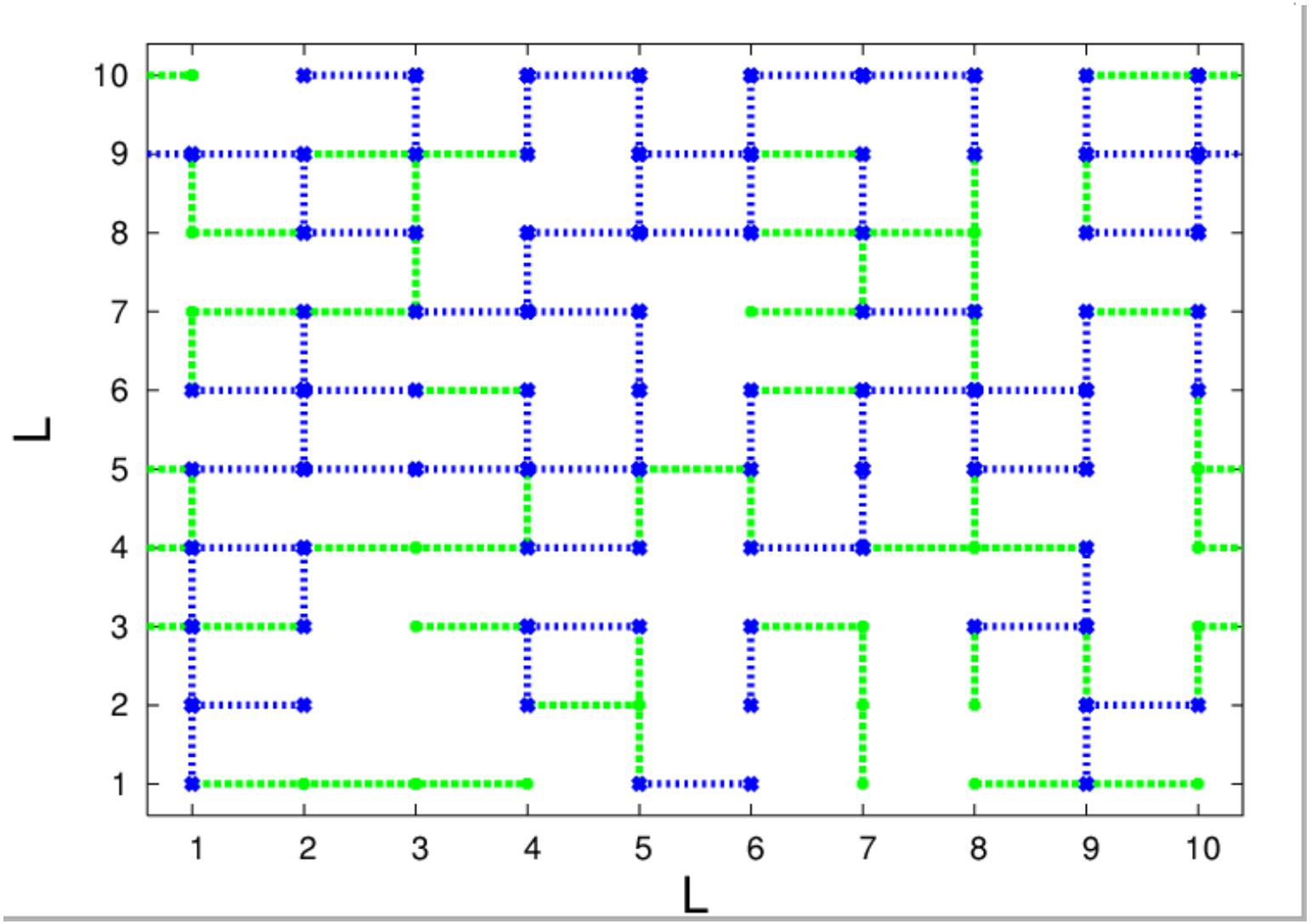
$\Rightarrow \sigma(\Delta E) \sim L^y$



⇨ Low Energy Excitations (like "small oscillations")

Lattice Spin Glasses (at $T=0$):

Defect-Energy:

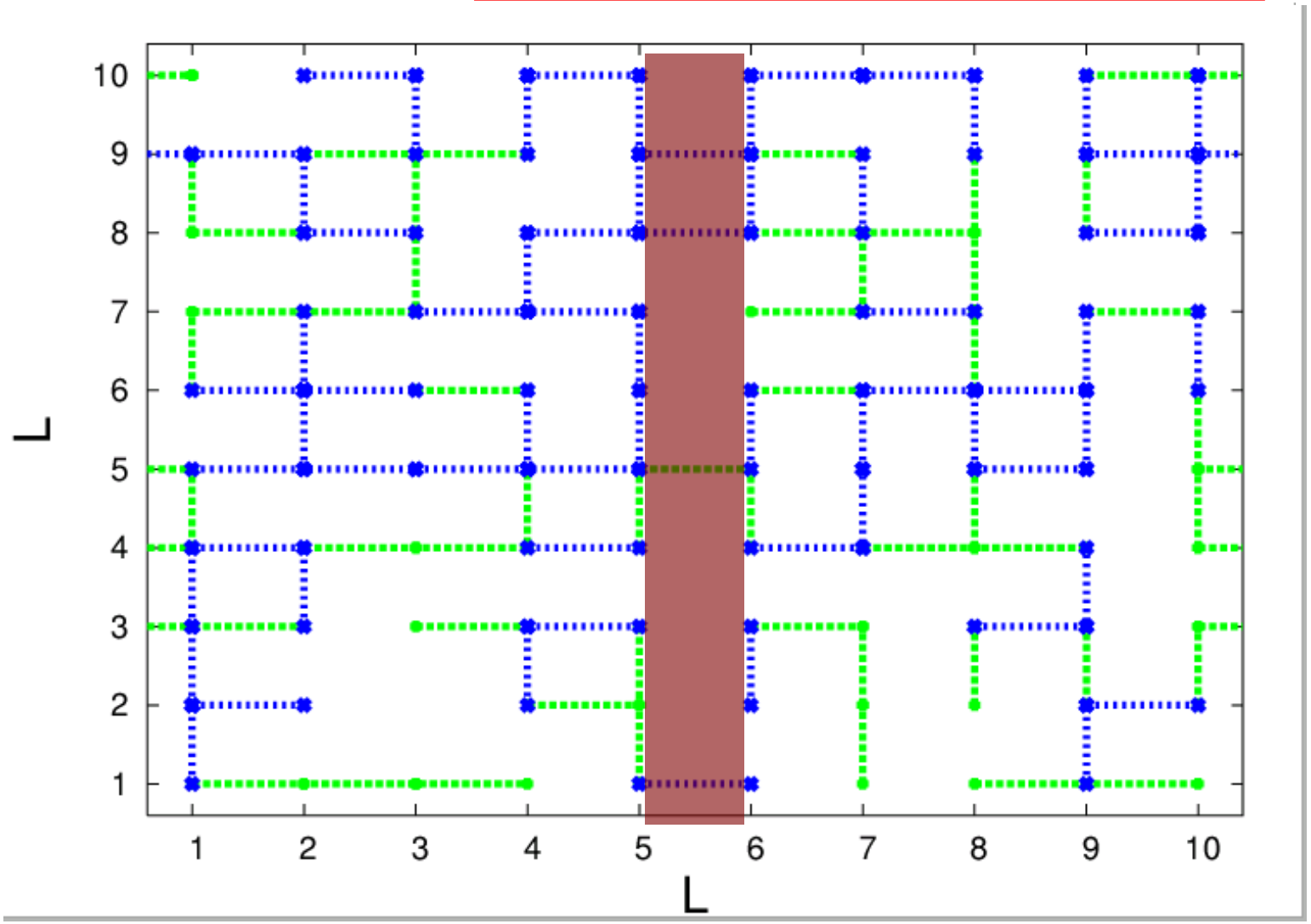


⇨ Low Energy Excitations of bond-diluted Lattices

Lattice Spin Glasses (at $T=0$):

Defect-Energy:

Collect $\sigma(\Delta E) \sim L^y$



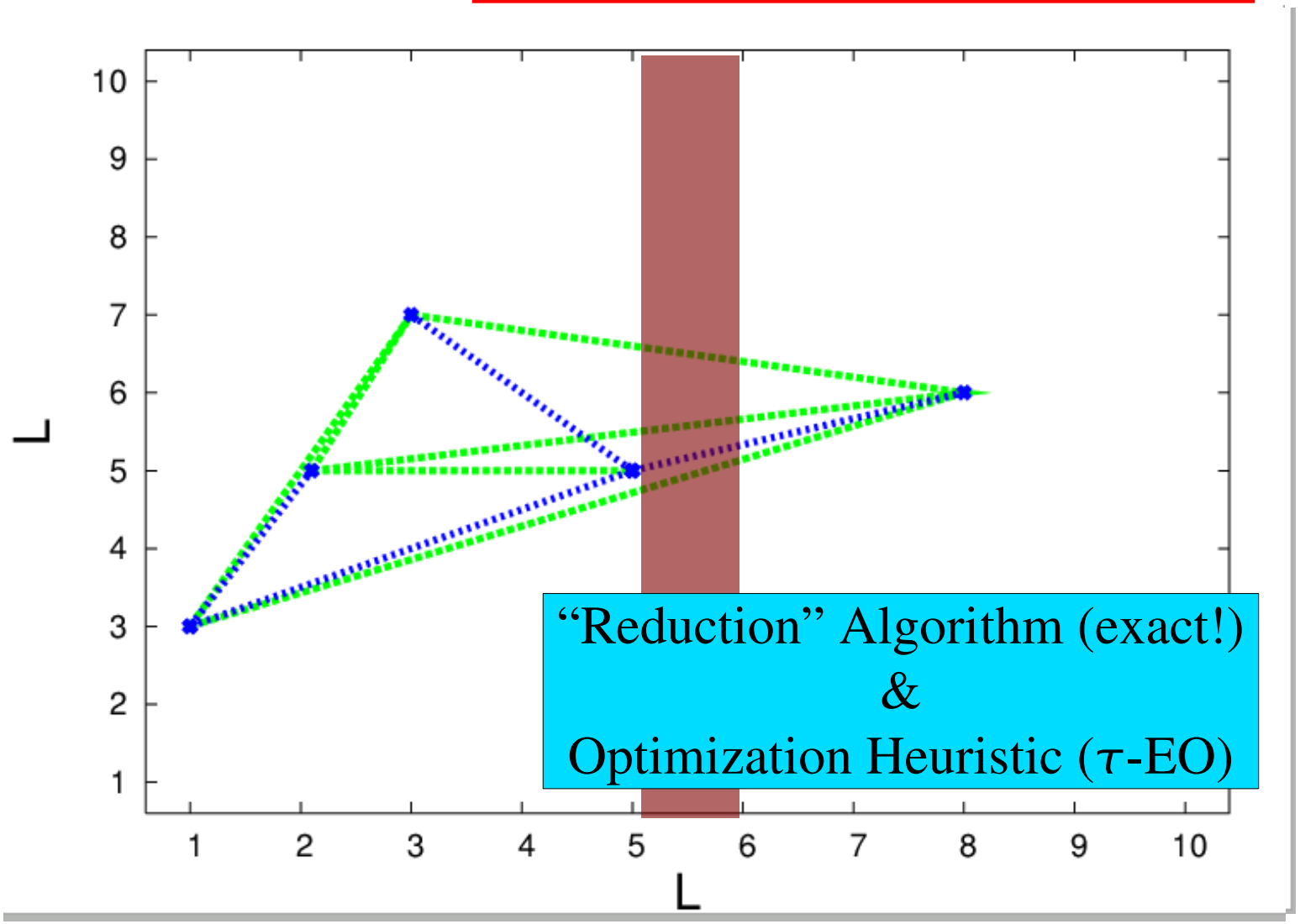
Before:
100 Spins

⇨ Low Energy Excitations of bond-diluted Lattices

Lattice Spin Glasses (at $T=0$):

Defect-Energy:

Collect $\sigma(\Delta E) \sim L^y$



Before:
100 Spins

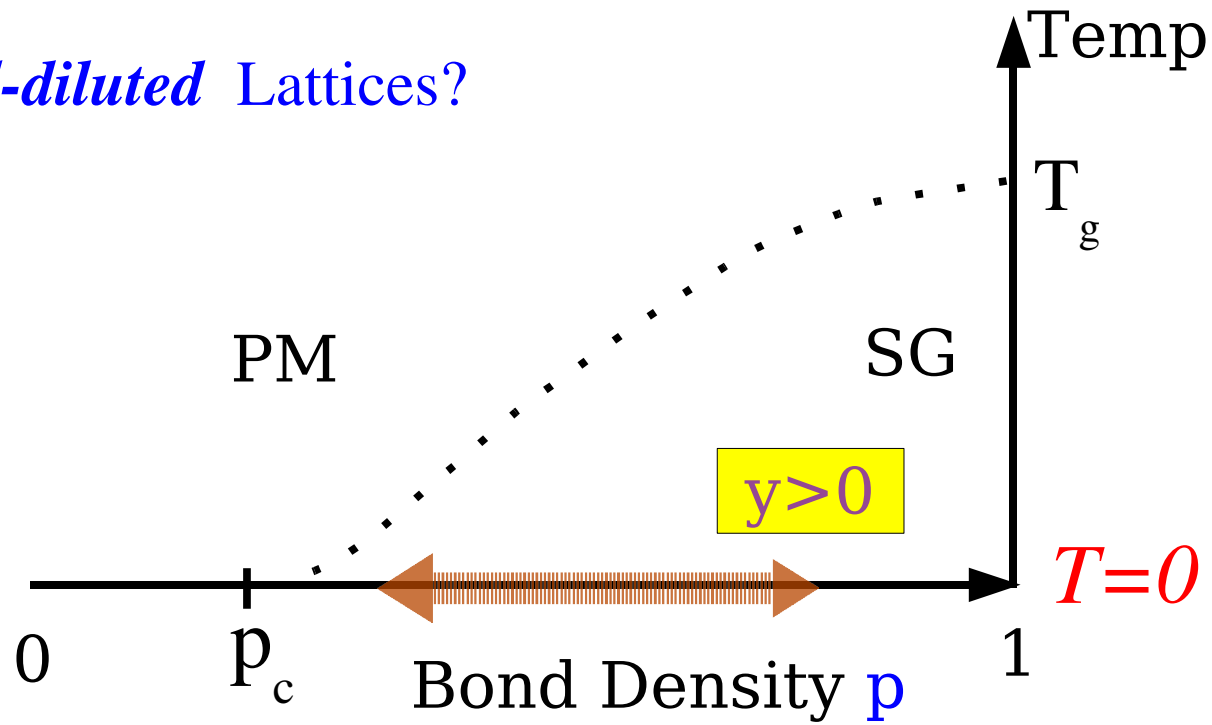
After:
5 Spins

⇨ Low Energy Excitations of bond-diluted Lattices

Lattice Spin Glasses (at $T=0$):

Defect-Energy: Measure "Stiffness": $\sigma(\Delta E) \sim L^y$

How ***bond-diluted*** Lattices?

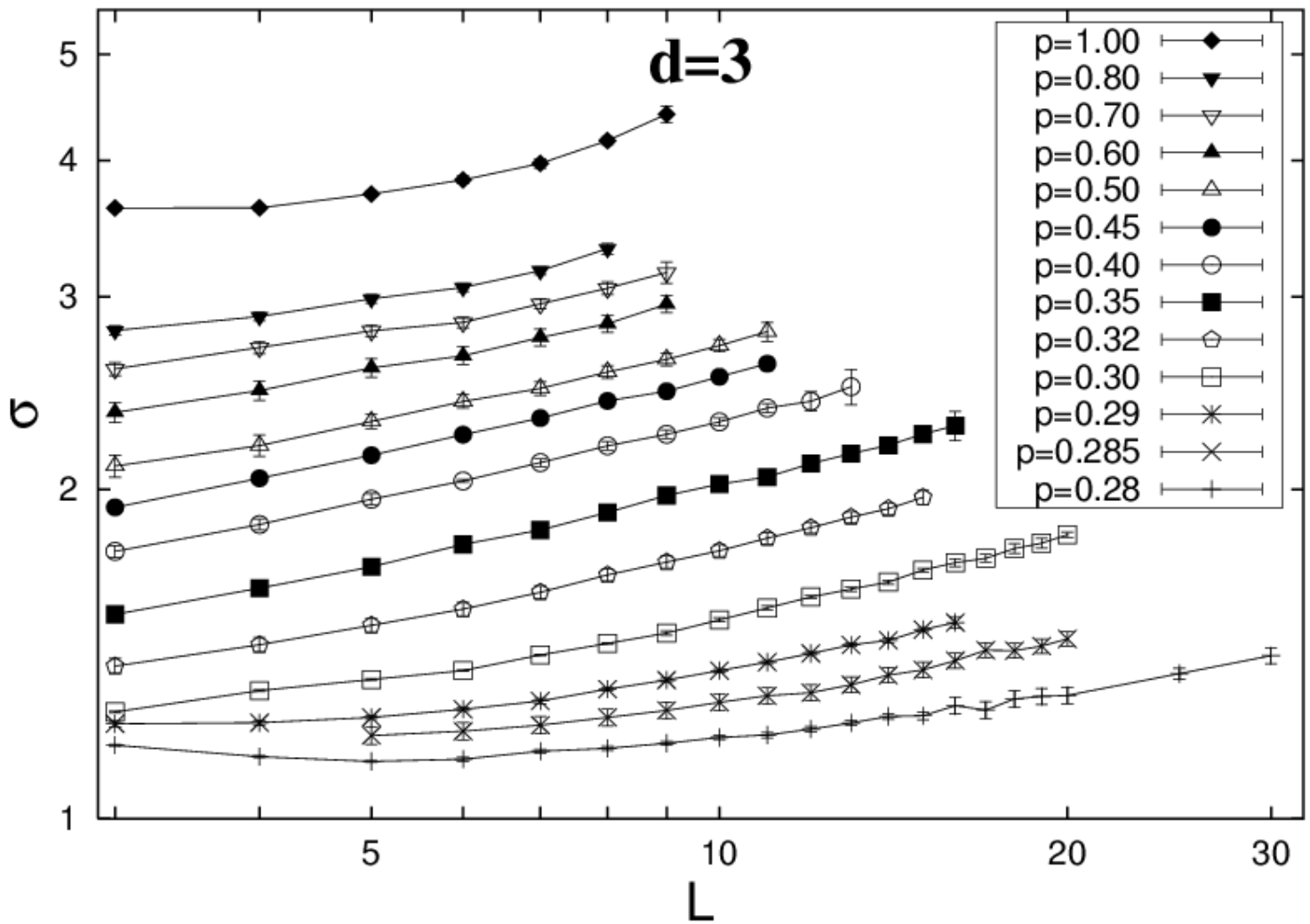


Why ***bond-diluted*** Lattices?

- Simpler Problem
- Larger Sizes L
- Better Scaling

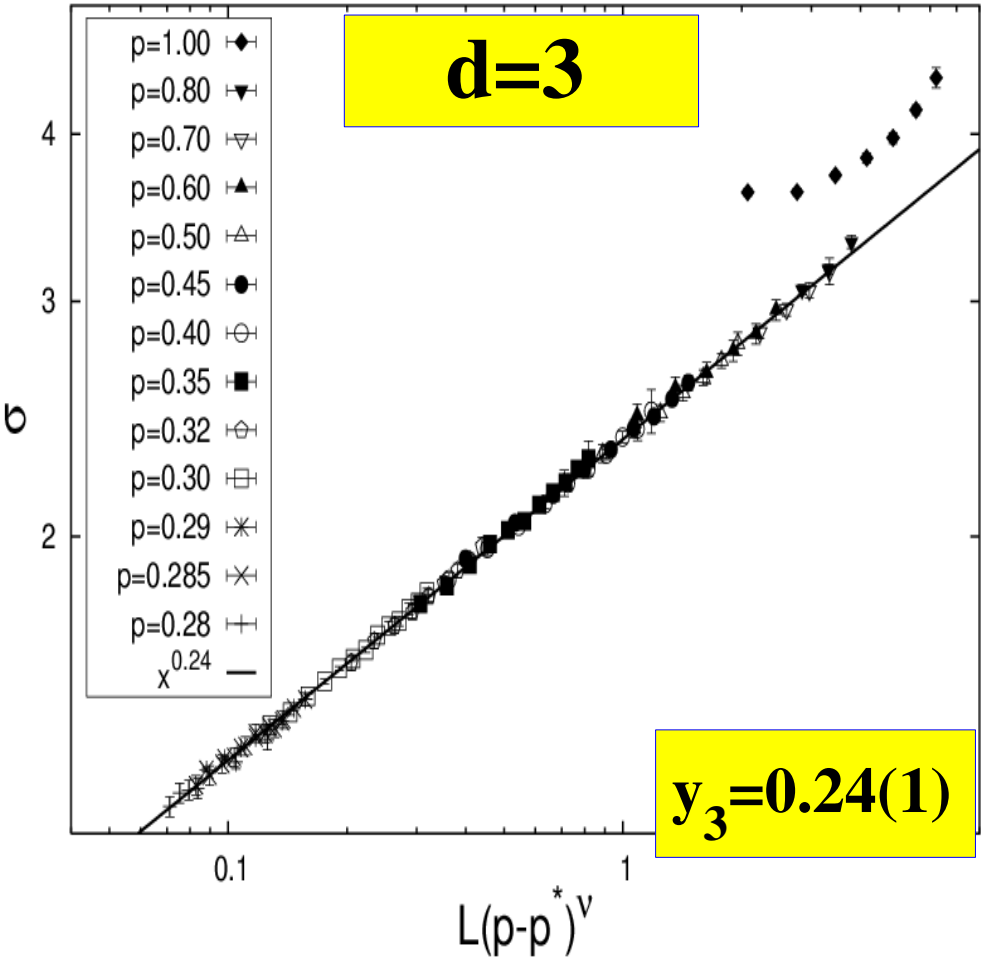
Defect-Energy of diluted Lattices:

- ±J-Glasses on Lattices of size L and density p.
- Defect-Energy σ(ΔE) with Reduction & Heuristic (τ-EO).



Stiffness Exponent *y* for Lattice Glasses:

"Stiffness": $\sigma(\Delta E) \sim L^{y}$



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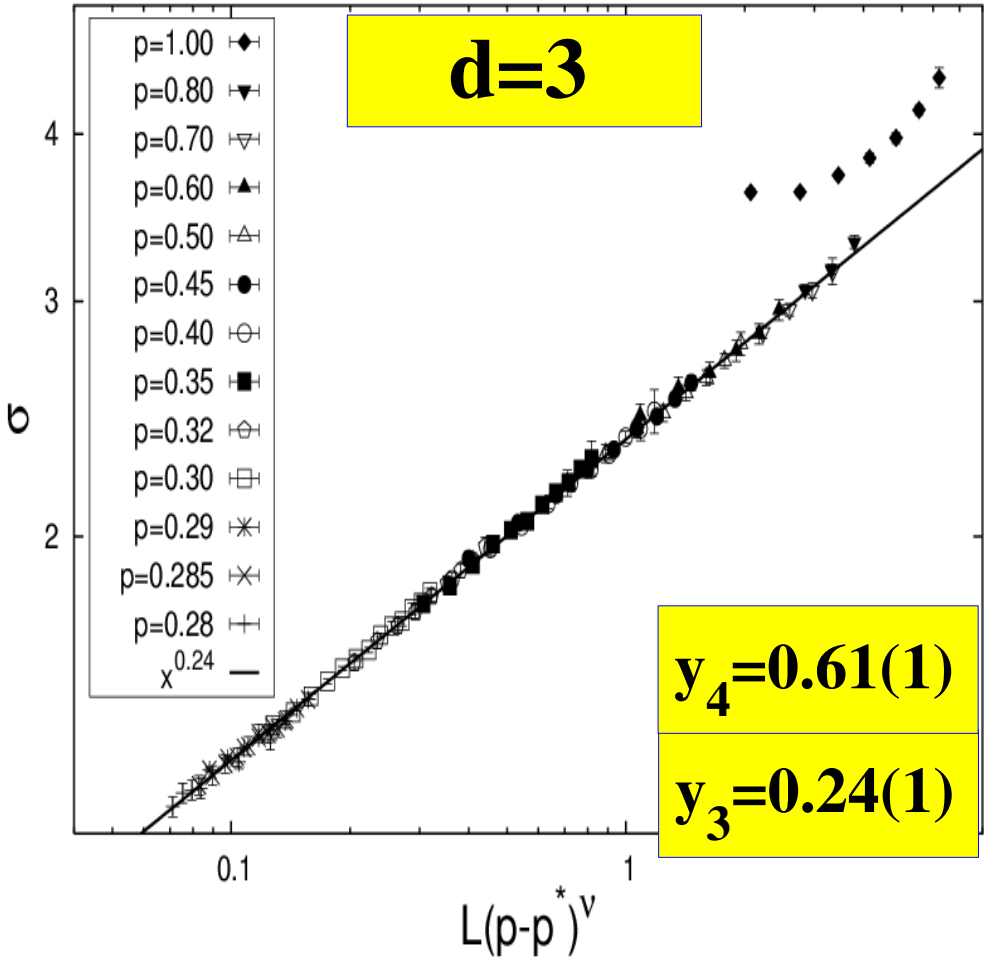

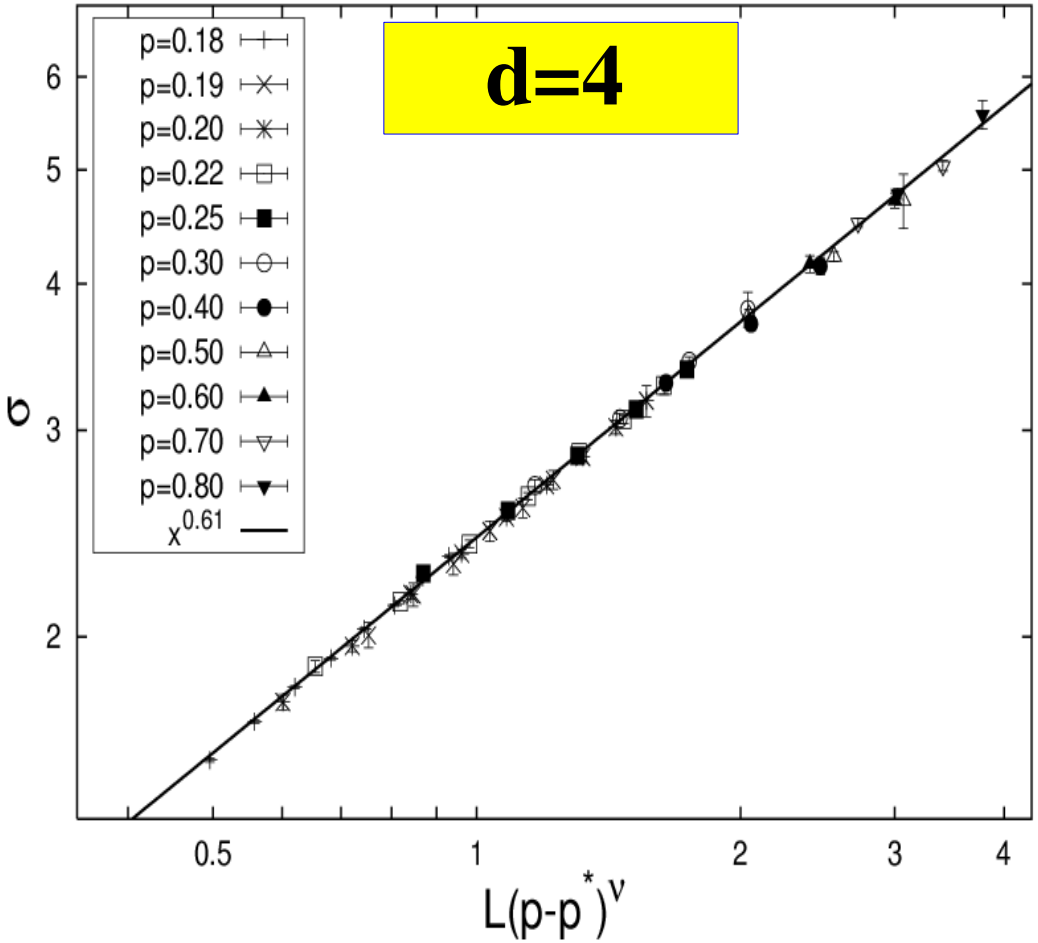

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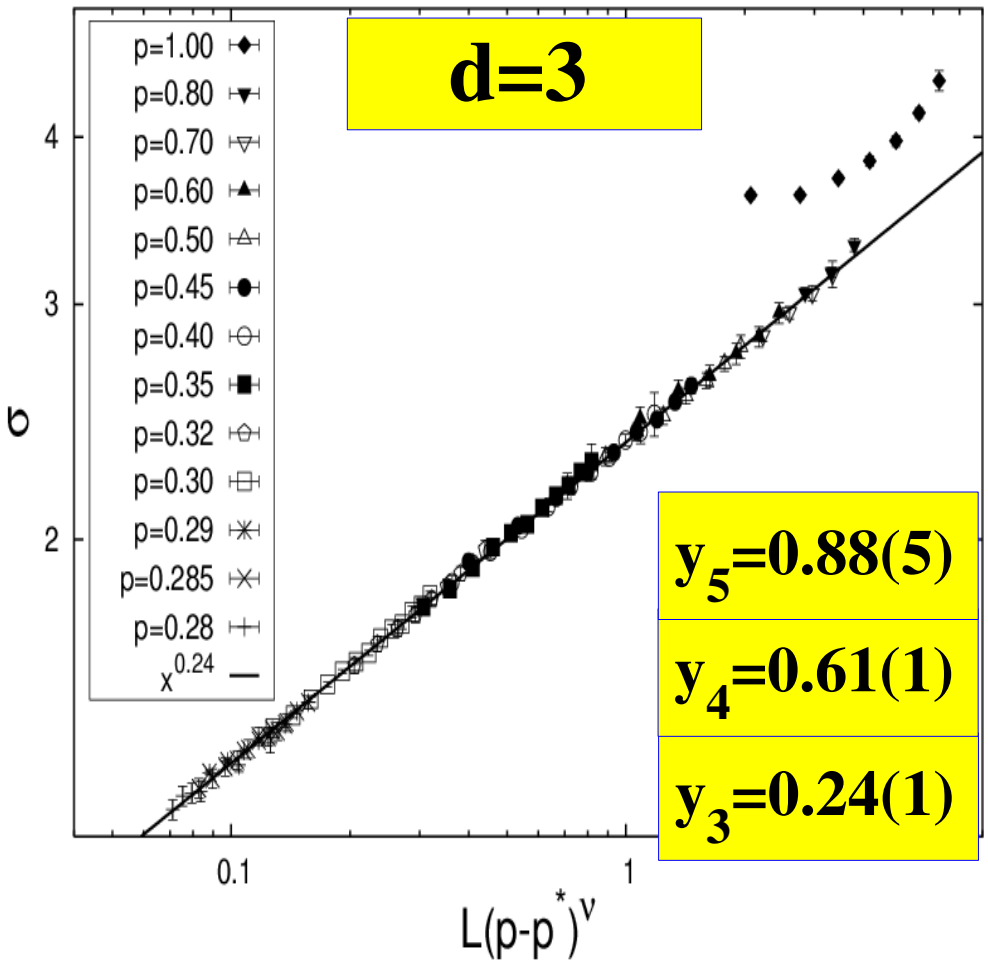
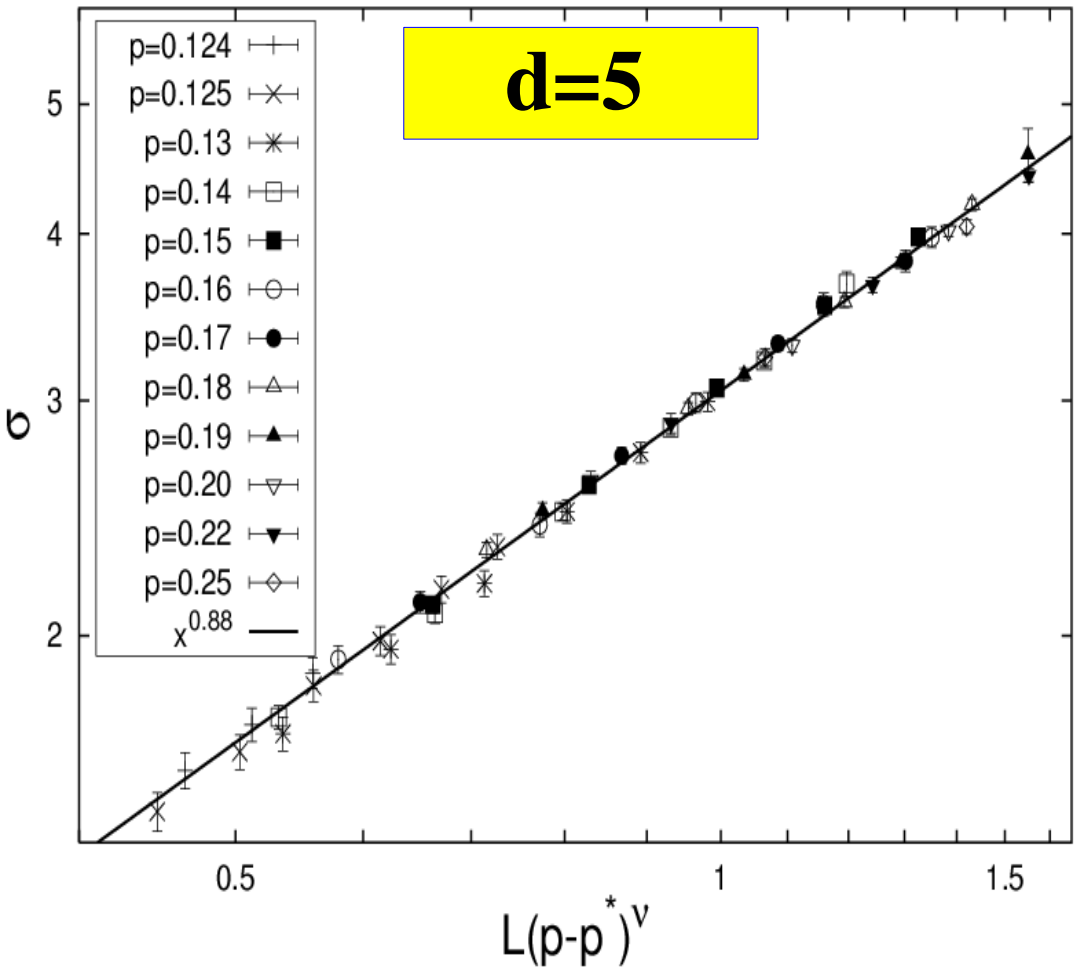


y_5=0.88(5)

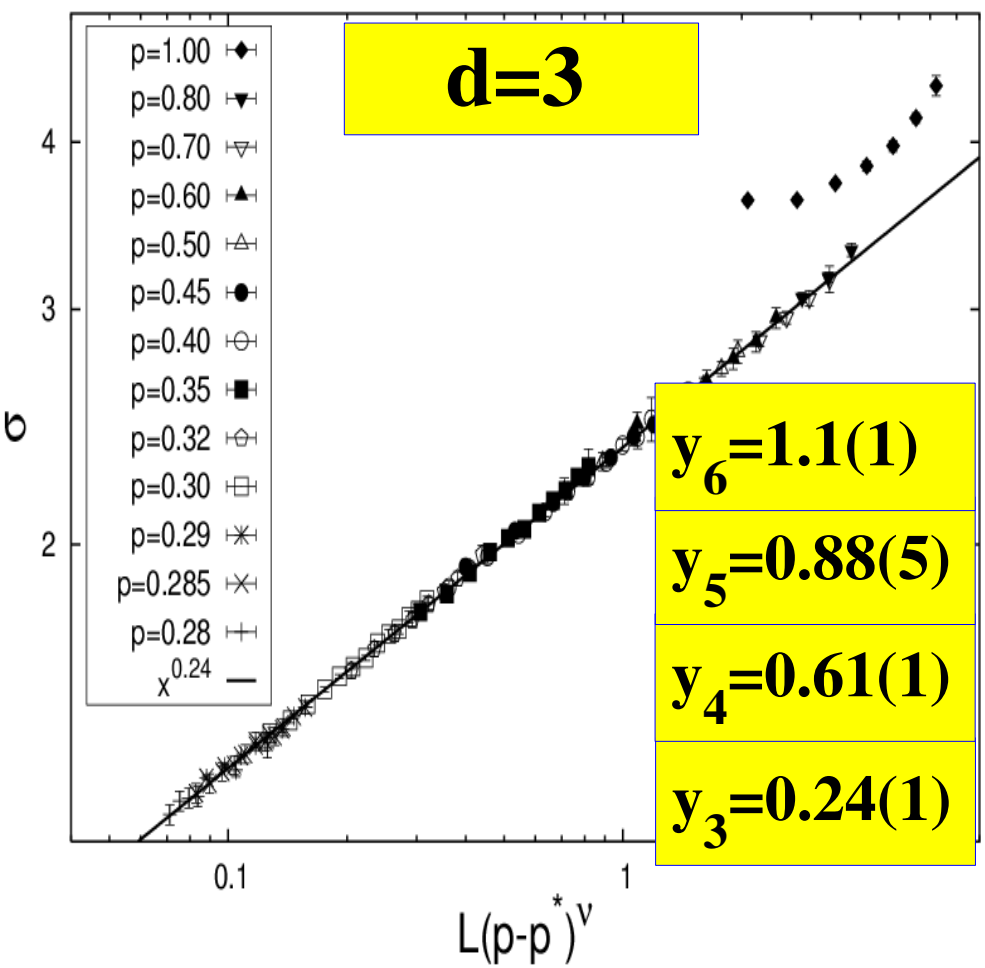
y_4=0.61(1)

y_3=0.24(1)



Stiffness Exponent *y* for Lattice Glasses:

"Stiffness": $\sigma(\Delta E) \sim L^{y}$

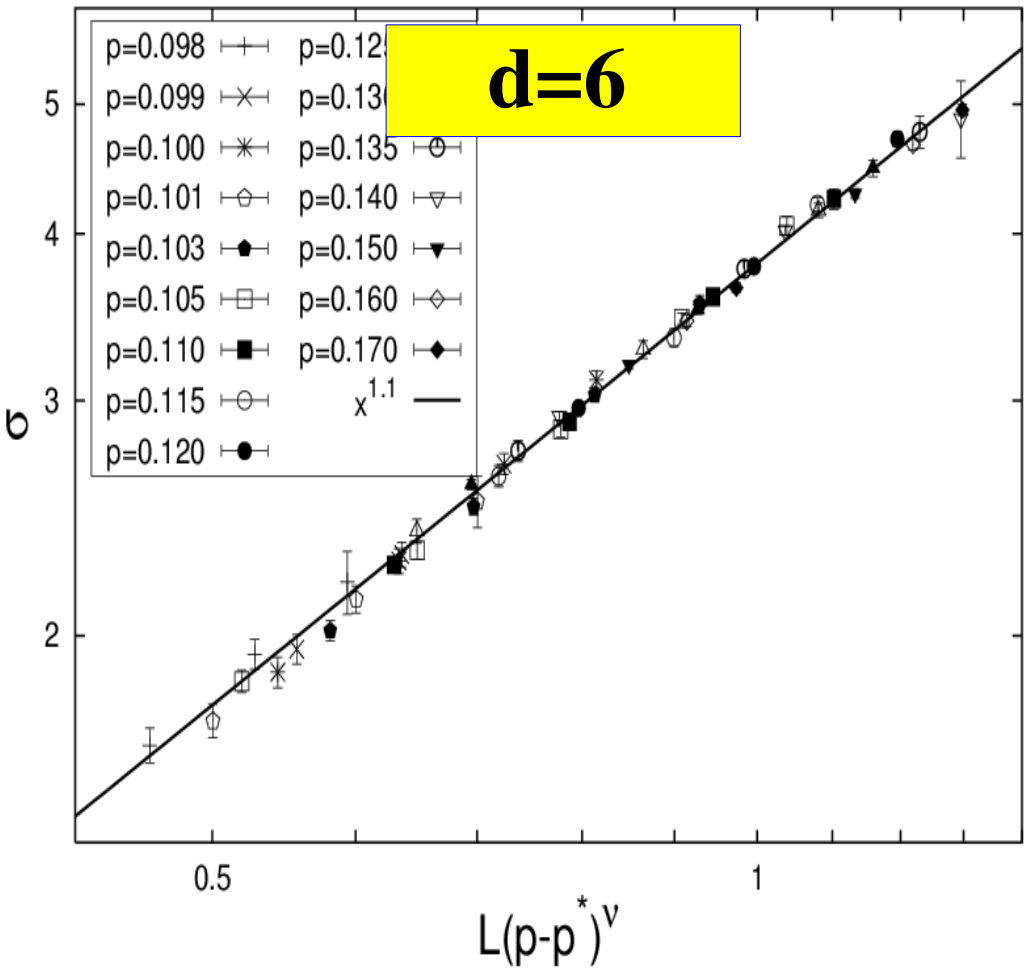


y_6=1.1(1)

y_5=0.88(5)

y_4=0.61(1)

y_3=0.24(1)



Stiffness Exponent *y* for Lattice Glasses:

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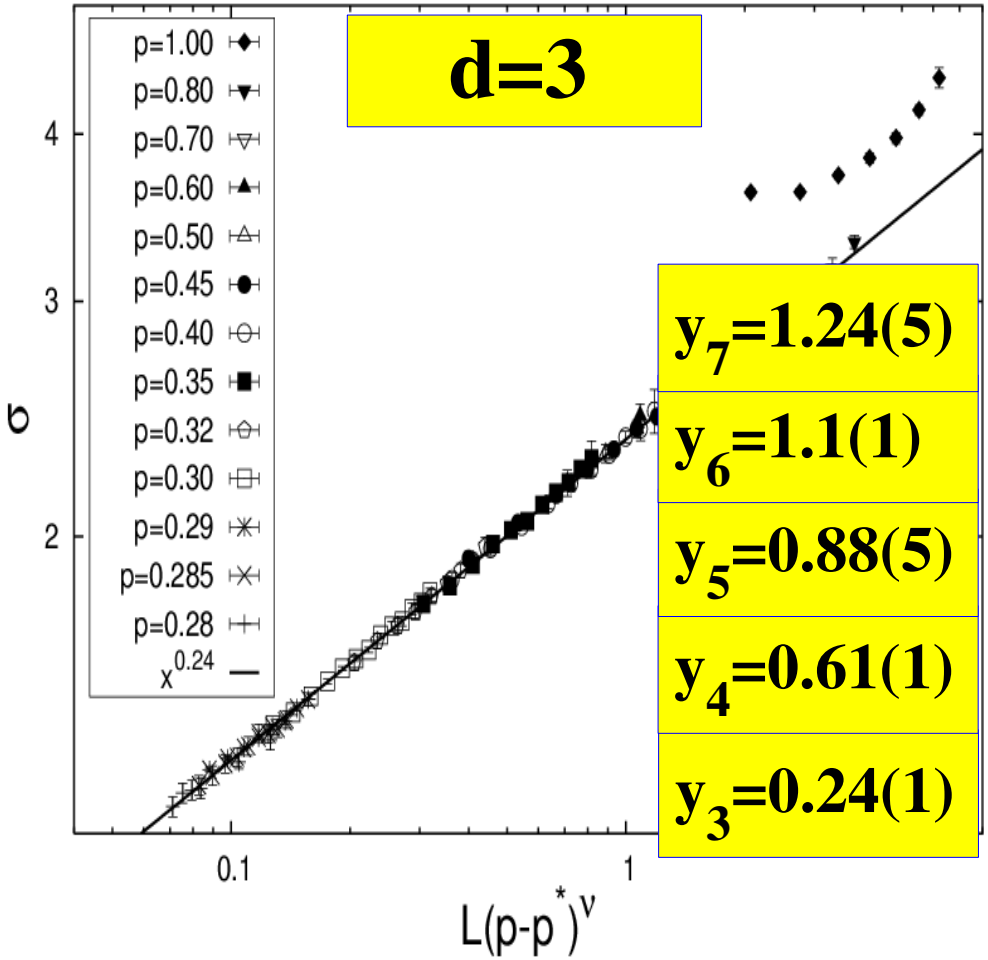


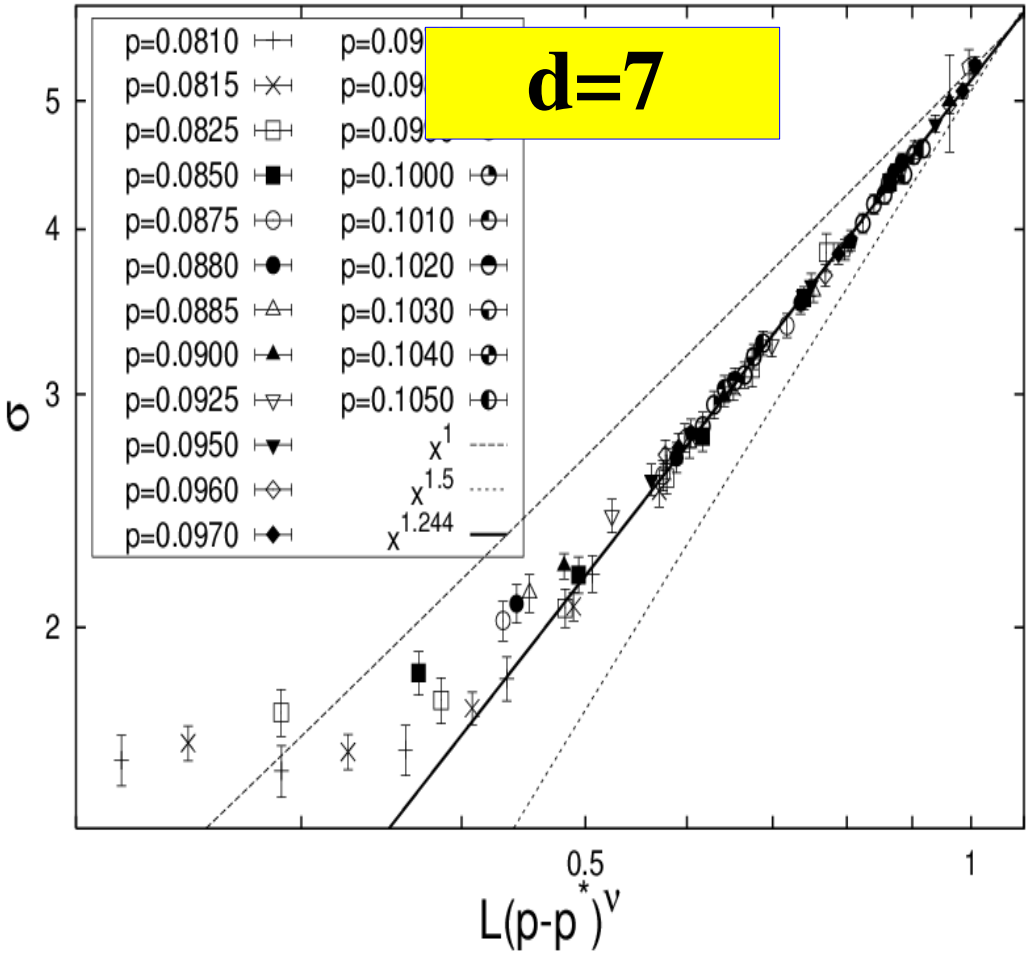
y_7=1.24(5)

y_6=1.1(1)

y_5=0.88(5)

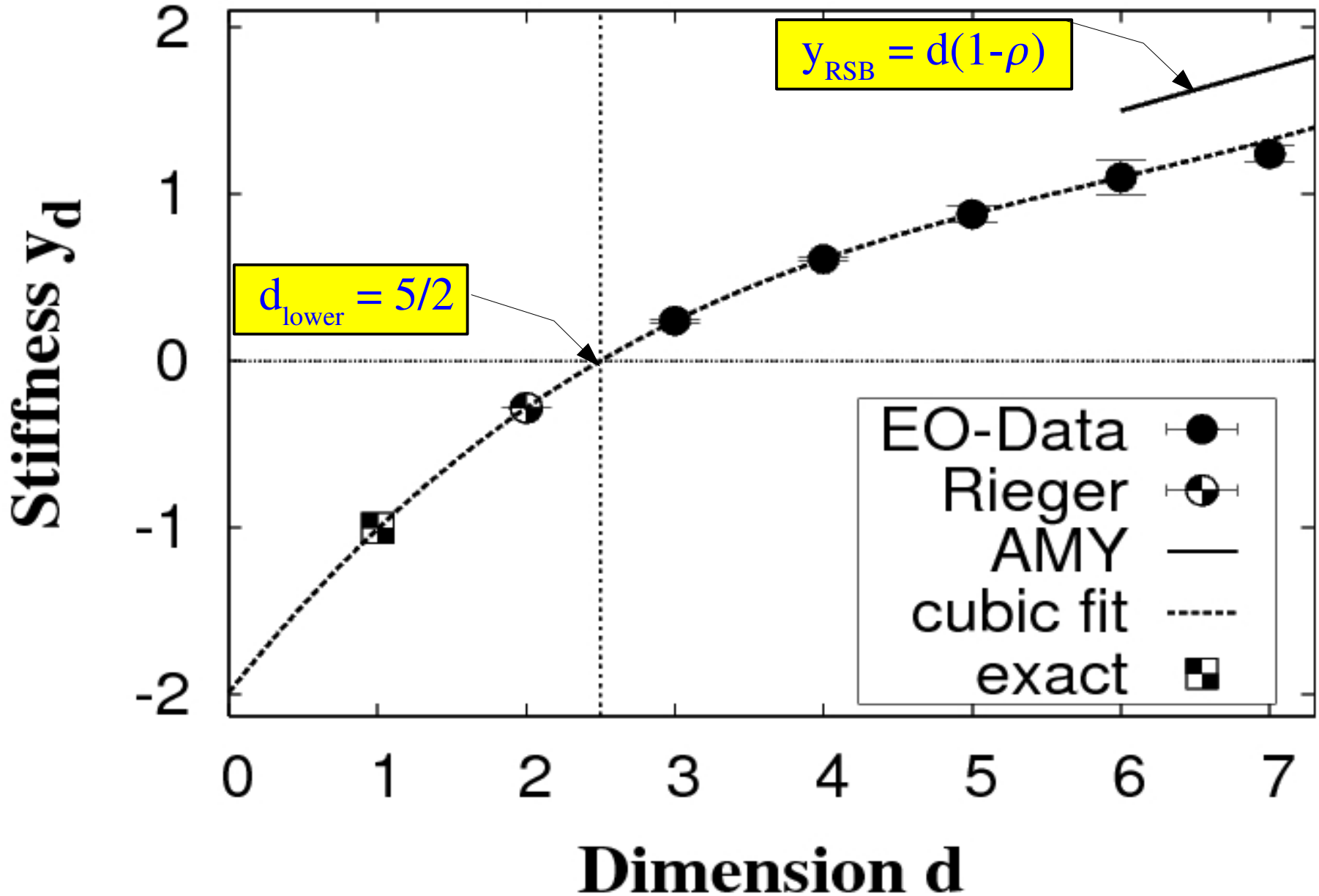
y_4=0.61(1)

y_3=0.24(1)



Comparing with Theory:

"Stiffness": $\sigma(\Delta E) \sim L^y$



Other Evidence for d_l=5/2:

- **From Theory:** (Franz,Parisi&Virasoro, J. Phys. I **4**,1657,'94) Effective Mean Field calculation near T_g, where Replica Symmetry Breaking (RSB) disappears (ie. $T_g \to 0$) for $d_l = 5/2$.

- **From Numerics:**

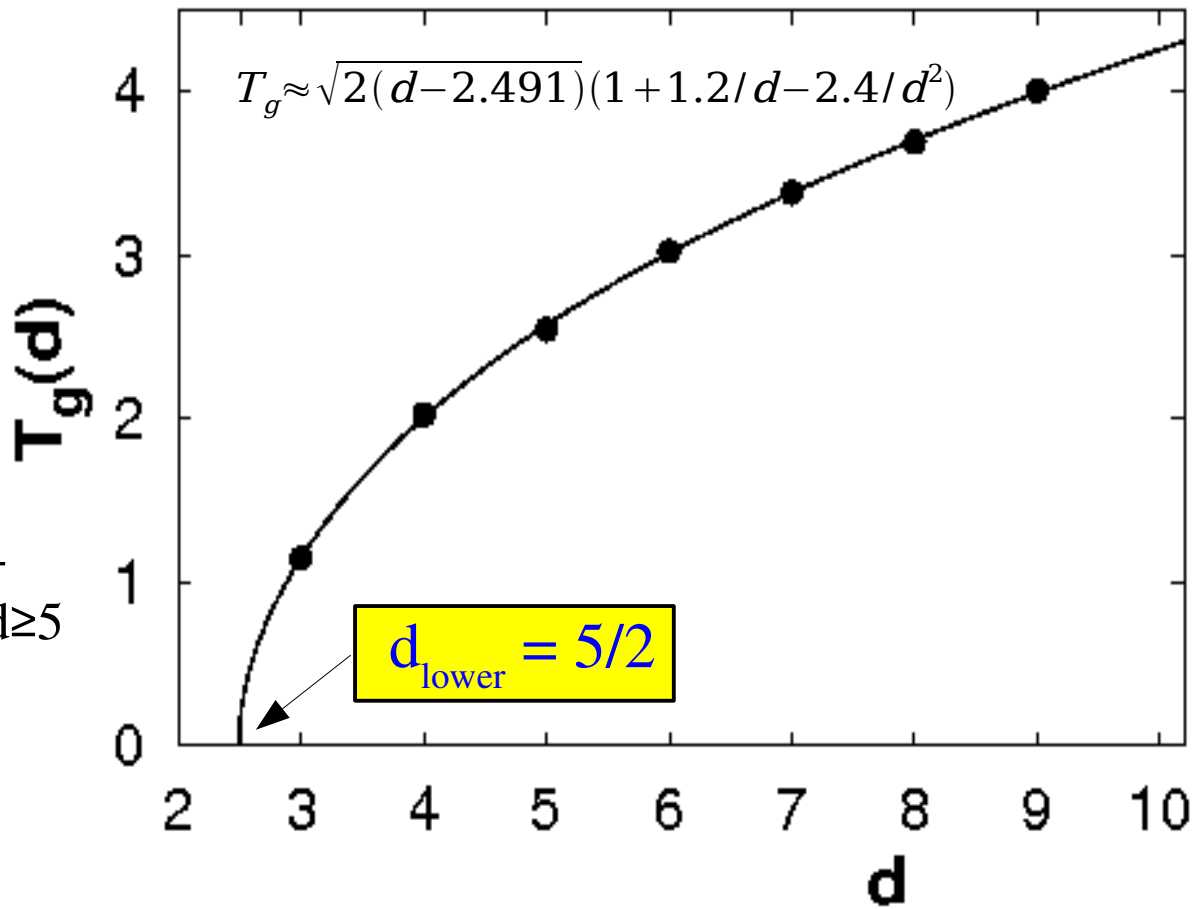
Know:

$T_g \approx \sqrt{2d}$ $\quad (d \to \infty)$

$T_g \approx \sqrt{d - d_l}$ $\quad (d \to d_l)$

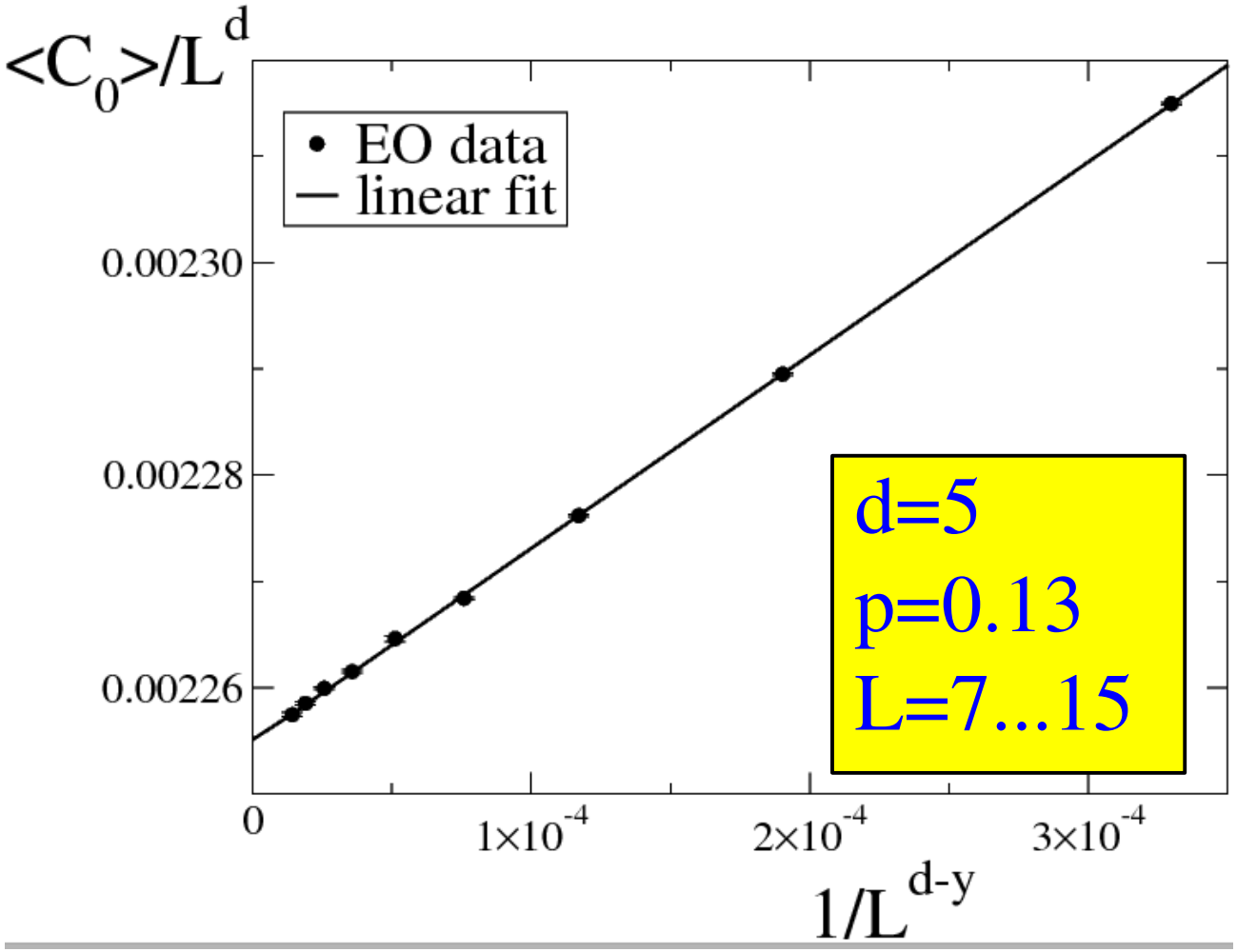
Data from:

- MC (Ballesteros et al) for d=3,4
- High-T Series (Klein et al) for d≥5



Corrections to Scaling in Spin Glasses:

Ground State Energy: $E(L) \sim e_0 L^d + A L^y \quad (L \rightarrow \infty)$



Comprehensive View on Spin Glasses:

A Set of Exponents:

1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$

- In **EA**: $\rho = ½$ (Wehr&Aizenman → exact!)

2) Distribution $P(\Delta E_0)$, width $\sigma(\Delta E_0) \sim N^{y/d} = L^{y}$

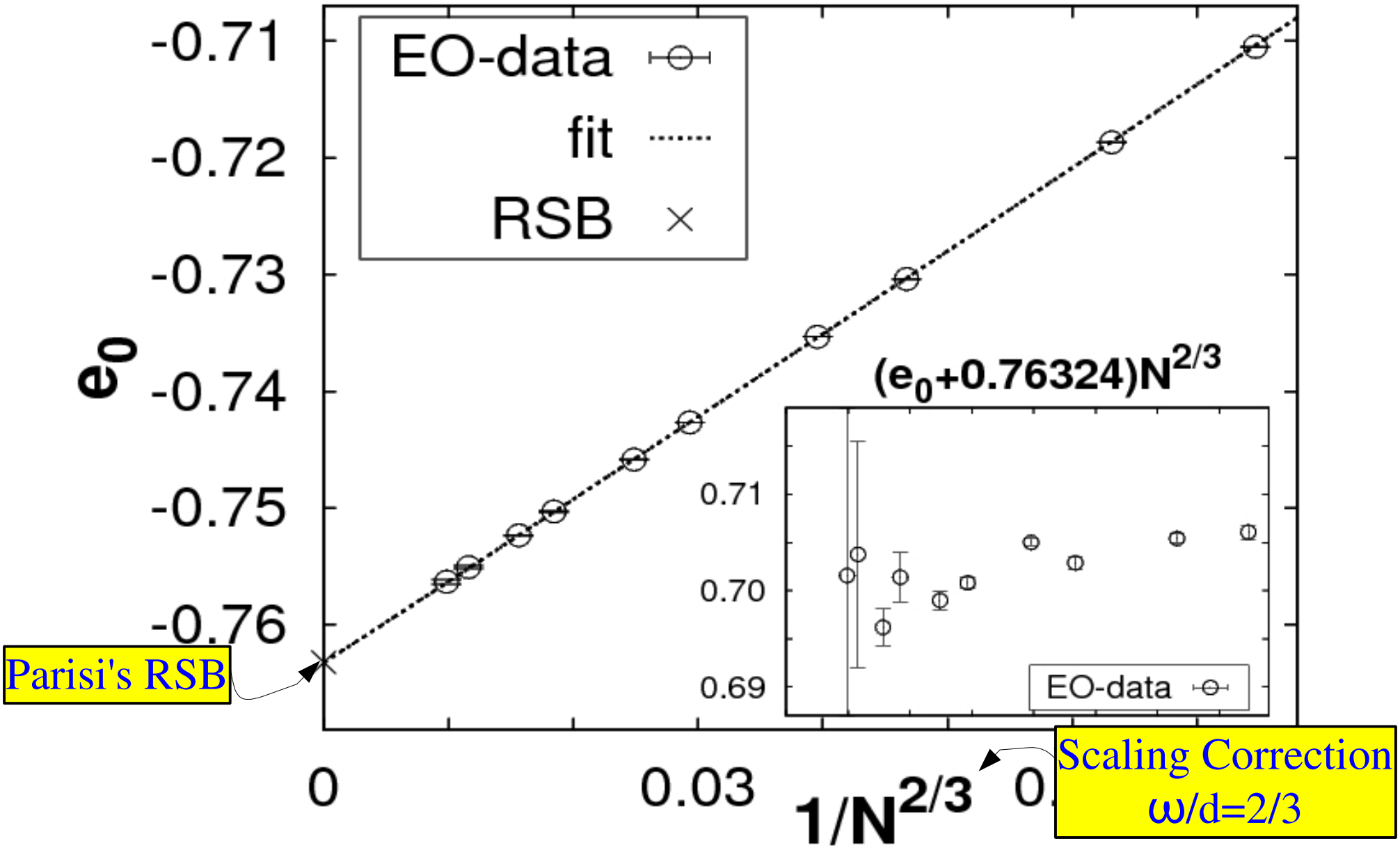
- In **EA**: $y \approx 0.24,\ldots,1.2$ for $d=3,\ldots,7$

3) Corrections-to-Scaling : $e_0(N)-e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$

- In **EA**: $\omega/d = 1 - y/d$

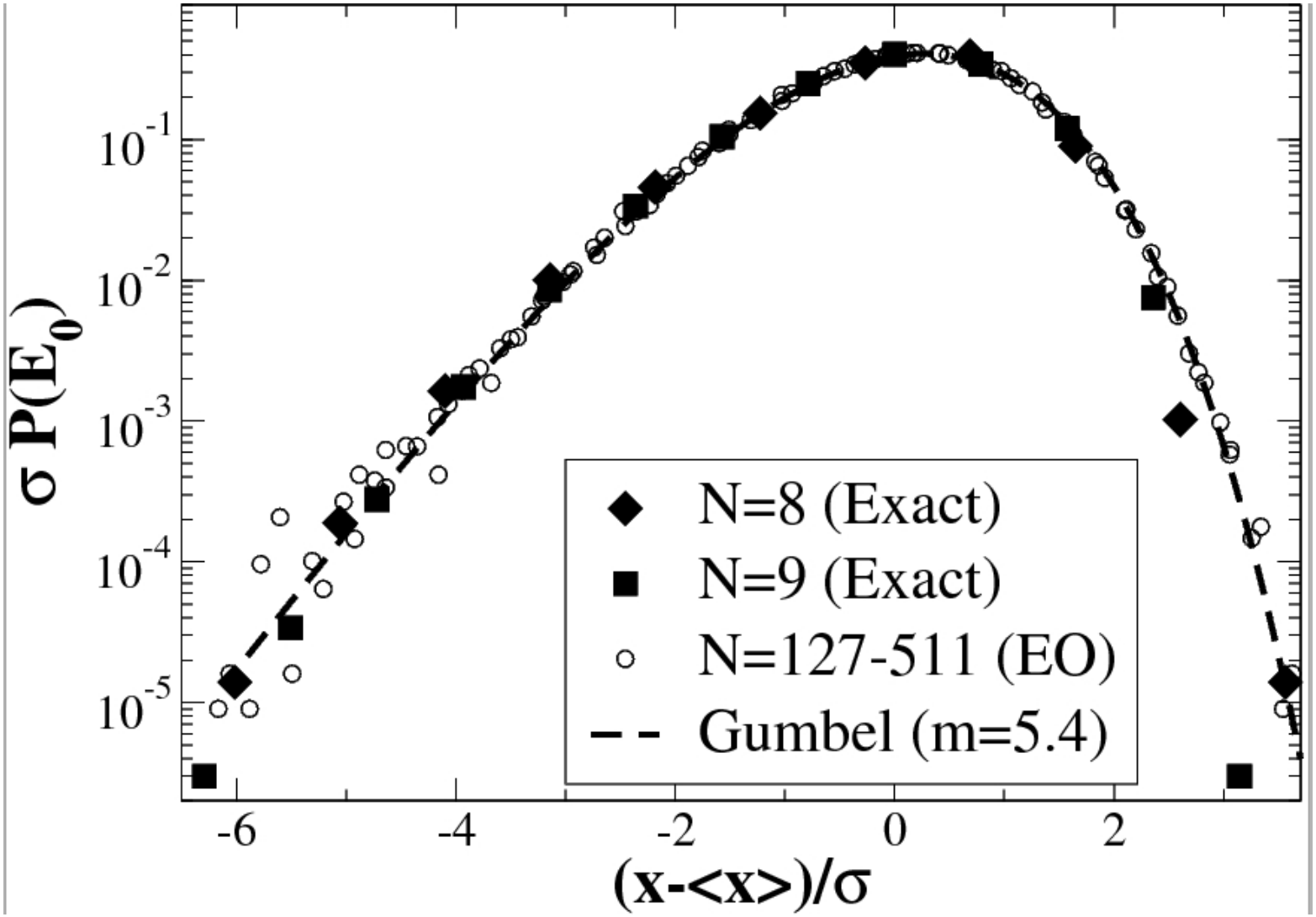
τ-EO for Sherrington-Kirkpatrick (SK):

- Mean-Field ($d \to \infty$) Spin Glasses:



Sherrington-Kirkpatrick (SK) at T=0:

- PDF for E_0 from ***exact*** Enumeration:



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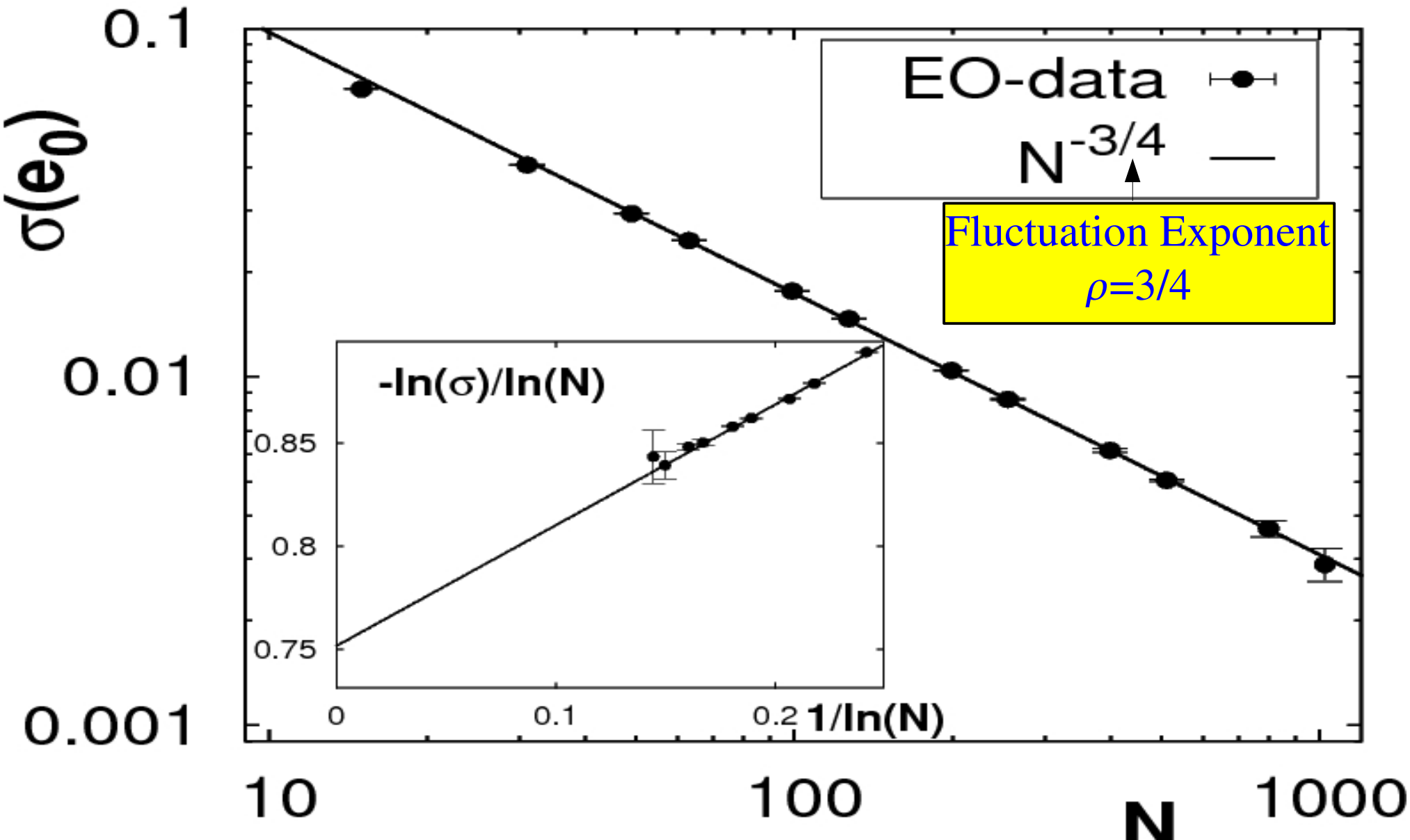
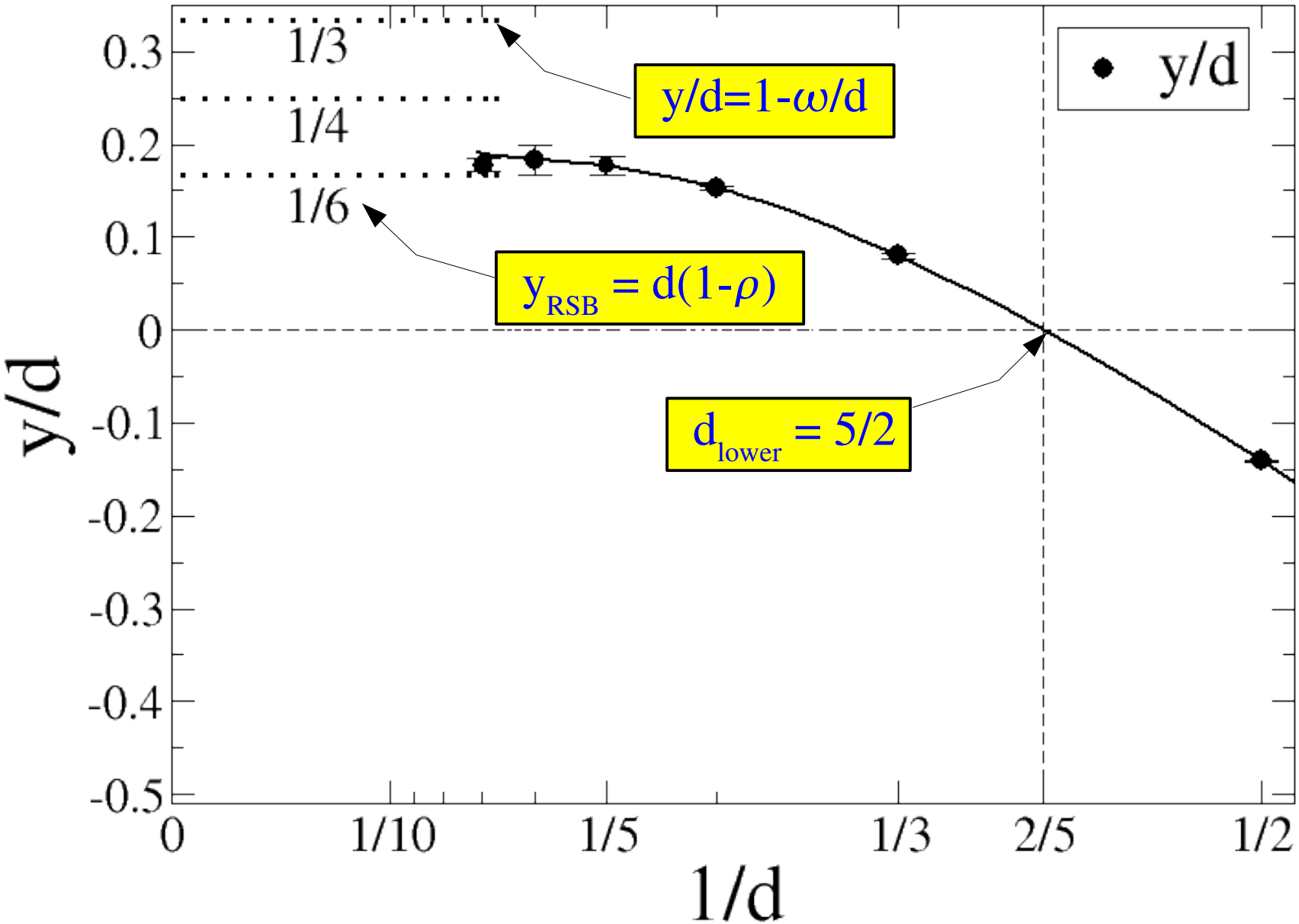


Comparing with Theory:

"Stiffness": $\sigma(\Delta E) \sim L^y$



Comprehensive View on Spin Glasses:

1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$

- In **EA**: $\rho = ½$ (Gaussian)
- In **SK**: $\rho \approx ¾$ (Highly Skewed)

2) Distribution $P(\Delta E_0)$, width $\sigma(\Delta E_0) \sim N^{y/d} = L^{y}$

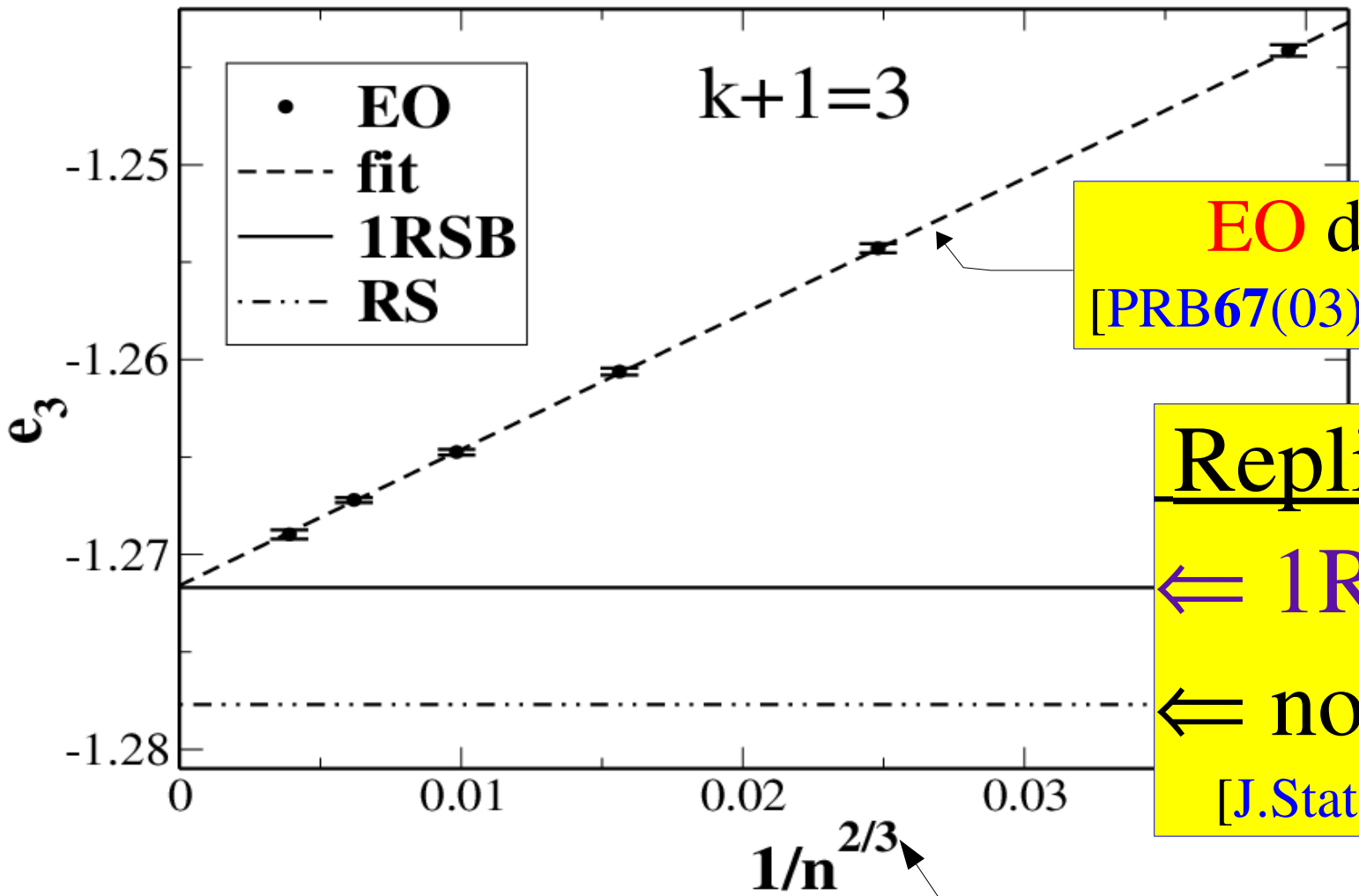
- In **EA**: $y \approx 0.24,\ldots,1.2$ for $d=3,\ldots,7$
- In **SK**: $y/d = 1-\rho \rightarrow 1/4$, too high for **EA** at $d \rightarrow \infty$

3) Corrections-to-Scaling : $e_0(N)-e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$

- In **EA**: $\omega/d = 1 - y/d$
- In **SK**: $\omega/d \approx 2/3 \neq 1 - y/d$

τ-EO for Bethe Lattices:

EO for 3-connected Bethe Lattice Glass w/ Replica Sym. Breaking:



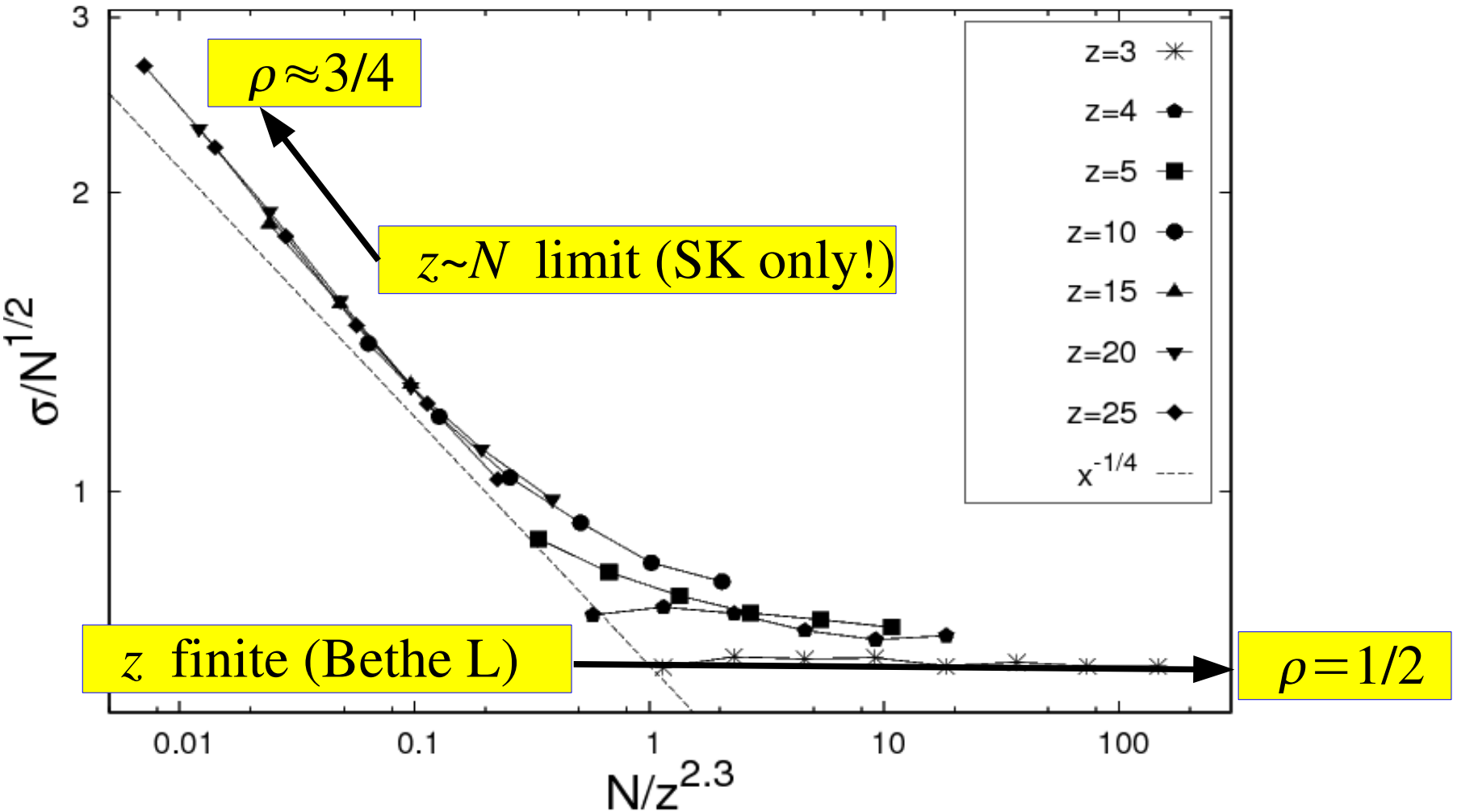
EO data,
[PRB**67**(03)060403R]

Replica Theory:
⇐ 1RSB,
⇐ no RSB,
[J.Stat.Phys.**111**(03)1]

Scaling corrections

τ-EO for Bethe Lattices:

GS variance for z-connected Bethe Lattice Glass:



Conclusions:

- Bond-Diluted Lattices:
  - Reduction of dilute Lattices => reach larger L
  - Finite-Size Scaling => extended Scaling Regime
  - Accurate Scaling Exponents (Stiffness y, CtS ω) ...
  - ...even in high dimensions
  - Allows Comparison with MFT
  - Allows Prediction of lower critical dimension $d_l=5/2$.

Outlook:

- Reduction Algorithm:
  - Determining $T_g \sim (p-p_c)^{\phi}$ → Experimentally Testable!
  - Determining fractal exponent of droplets.