Galactic Rotation

1. Objective

Use Doppler shift to determine the rotational speed of the WHOLE ENTIRE GALAXY.

2. Introduction

2.1 The 21 cm line of neutral hydrogen

Neutral hydrogen can emit radio waves with a 21 cm wavelength and a frequency of 1420 MHz. This is a very long wavelength and a very low frequency, compared with the visible light of the Balmer series. You know that the lines of the Balmer series result from electrons dropping to the n = 2 energy level from higher energy levels. So where does the 21 cm wavelength come from?

The ground state of the hydrogen atom can actually be split into two energy levels: one in which the proton and electron have parallel spins (shown in Figure 1), and one in which the spins are antiparallel. The parallel-spin state has a higher energy, and when the atom transistions from parallel spins to antiparallel spins, light with the 21 cm wavlength is released.

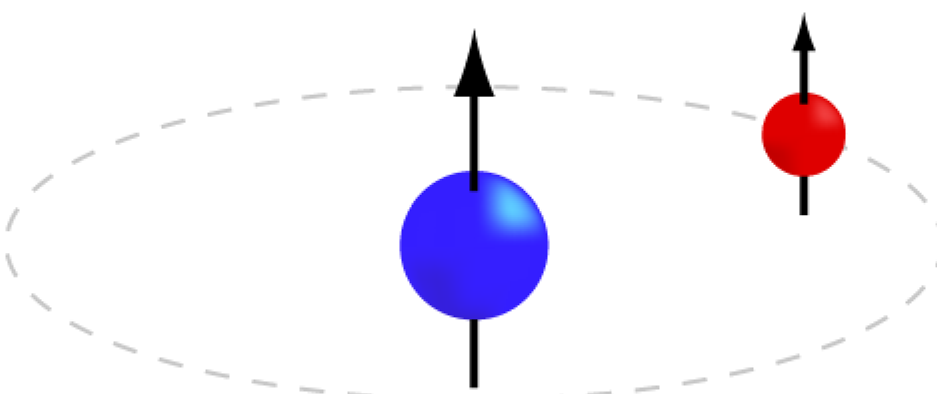

Figure 1 (from http://en.wikipedia.org/wiki/File:HydrogenLineAntiParallel.png). Parallel spins in the ground state. When the atom transitions to the lower-energy state of antiparallel spins, light with a 21 cm wavelength is released.

The energy-level splitting due to the relative orientation of proton and electron spins is called hyperfine splitting. Why hyperfine? Fine (structure) splitting is due to the relative orientation of electron spin and orbital angular momentum. Since there's no orbital angular momentum in the ground state, the ground state is split only because of hyperfine structure. Figure 2 shows fine and hyperfine structure splitting. The quantum numbers have the following meanings: I is proton angular momentum, J is total electron angular momentum (orbital plus spin), and F is total atomic angular momentum (electron plus proton).

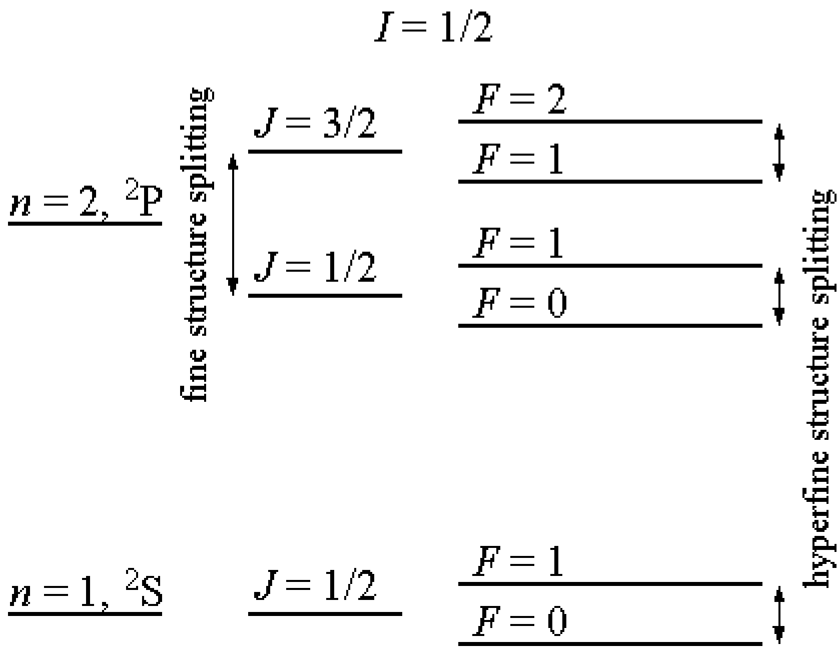

Figure 2 (from http://en.wikipedia.org/wiki/File:Fine_hyperfine_levels.png).

Hydrogen is the most abundant element in the universe, and interstellar clouds of hydrogen are cheerfully emitting the 21 cm line (frequency f_0 = 1420 MHz). However, if we're moving relative to the hydrogen cloud (and we almost certainly are), Doppler shift will affect the measured frequency. If the hydrogen cloud and the observer are moving away from each other with a total line-of-sight speed of v_{away}, the Doppler shift satisfies

$$-c\Delta f/f_0 = v_{away}. \tag{1}$$

This is a redshift (negative Δf).

2.2 Astronomical coordinate systems

Most of the hydrogen in the galaxy is in the galactic plane (the disk of the Milky Way), so it's convenient to use galactic coordinates, shown in Figure 3. If we're standing in a remote location on a starry night, we can look up and see the Milky Way as a great circle across the sky. This circle defines the galactic equator, b = 0. We will restrict our attention to points along the galactic equator (b = 0) so that we are looking along the disk of the Milky Way. It will be convenient to choose galactic longitudes in the first quadrant ($0° < \ell < 90°$).

It would be nice if we could type galactic coordinates into the electronics controlling our telescope. However, we need to give our telescope horizon coordinates, shown in Figure 4. Our telescope cannot face west; our azimuth can go no lower than about 5° and no higher than about 175°. The telescope's accessible altitudes range from about 10° to almost 90°. To minimize vulnerability to wind damage, we always "park" the telescope at the greatest possible altitude (~90°), and we don't use the telescope when wind speeds are excessive.

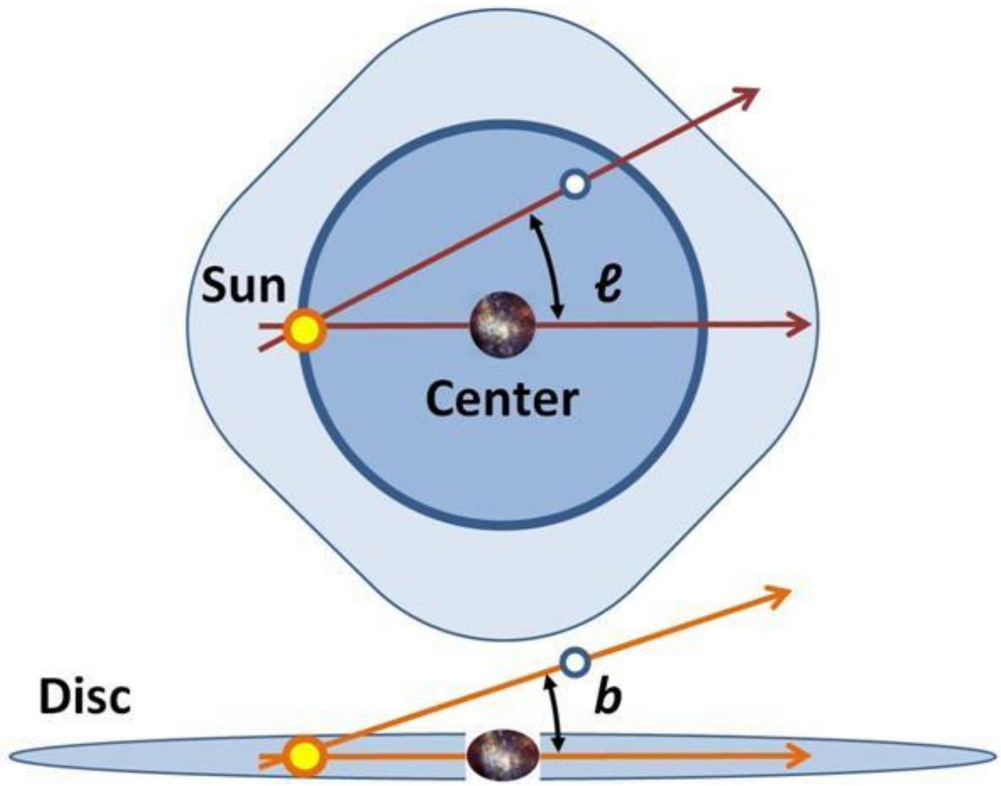

Figure 3 (from http://en.wikipedia.org/wiki/File:Galactic_coordinates.JPG). Galactic longitude (ℓ) and latitude (b).

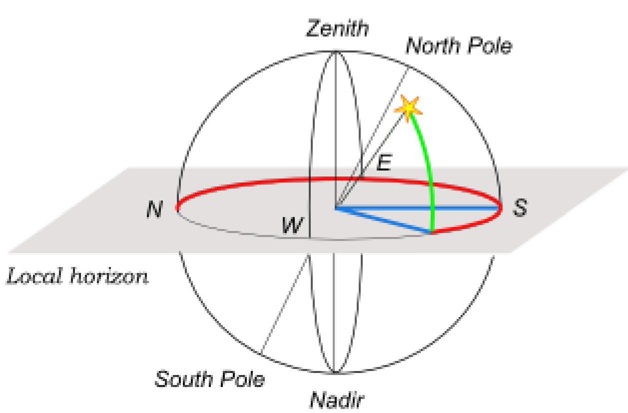

Figure 4 (from http://en.wikipedia.org/wiki/File:Horizontal_coordinate_system_2.png). Horizon coordinates. Altitude is angle shown in green (between the horizon and the point of observation). Azimuth is the angle shown in red.

It would be nice if we could convert from galactic coordinates directly to horizon coordinates. However, we have to convert to equatorial coordinates as an intermediate step. The two equatorial coordinates are declination (δ) and right ascension (α). Declination is analogous to latitude and is the angle between the equatorial plane (see Figure 5) and the line of sight (positive to the north, negative to the south). Right ascension is analogous to longitude. Longitudinal circles (passing through the north and south poles, projected into the sky) are called hour circles. Consider these hour circles to be fixed in the celestial sphere. Which hour circle do we choose for our longitudinal origin? We choose (half of) the hour circle passing through the sun at the vernal equinox. (As viewed from earth, the sun moves through the celestial sphere over the course of the year.) Right ascension is the angle in the equatorial plane between the hour circles of the vernal equinox and the observed object. It's positive to the east and typically measured not in degrees but in hours, with 15° equivalent to 1 hour. If the vernal equinox's hour circle ($\alpha = 0^h$) is directly overhead, then in 1 hour, the $\alpha = 1^h$ hour circle will be directly overhead.

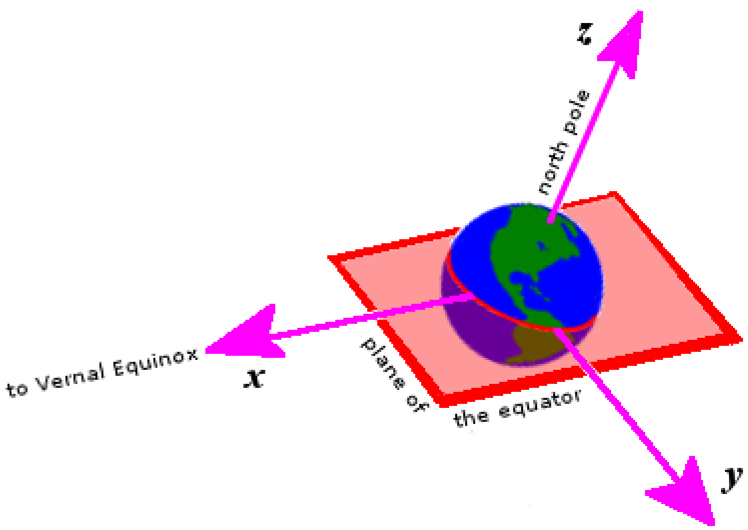

Figure 5 (from http://en.wikipedia.org/wiki/File:Ra_and_dec_rectangular.png). The equatorial plane and the direction toward the sun at the vernal equinox.

Since you need to convert a limited set of galactic coordinates (perhaps ℓ = 0°, 5°, … , 90°, all with b = 0°) into equatorial coordinates, your quickest option is probably to use on online calculator (http://hea.iki.rssi.ru/AZT22/ENG/cgi-bin/c_prec4.htm: enter l and b, enter year in either Start or Final Epoch, click Accept Galactical Coordinates) and tabulate the results. However, the corresponding horizon coordinates change with time! You need to predict when the desired locations in the galactic plane are in range of the telescope. Therefore, it's valuable to convert from equatorial to horizon coordinates in your own spreadsheet; you can enter different local sidereal times and instantly see the changes in all your horizon coordinates. Local sidereal time is given in the Spectracyber software. You can also find an online calculator to convert from equatorial to horizon coordinates.

To do the conversion yourself, use these formulas:

- Compute the hour angle $H = t - \alpha$, where t is local sidereal time.
- Compute the altitude $a = \sin^{-1}(\cos H \cos\delta \cos\varphi + \sin\delta \sin\varphi)$, where φ is our latitude.
- Compute azimuth A by making use of *both* of these formulas:

$\sin A = -\sin H \cos\delta / \cos a$ $\qquad$ $\cos A = (\sin\delta - \sin a \sin\varphi)/(\cos a \cos\varphi)$

Why do we need two formulas for A? A can be any angle, 0°-360°. However, arcsine always gives a result between -90° and 90°, and arccosine always gives a result between 0° and 180°. If, for example, A = 270°, then cosA = 0, $\cos^{-1}$(cosA)) = 90°, and A = 360° - $\cos^{-1}$(cosA)). The general trigonometric rules (not specific to astronomy) are

$A = \cos^{-1}(\cos A))$ if $\sin A > 0$

$A = 360° - \cos^{-1}(\cos A))$ if $\sin A < 0$

$A = \sin^{-1}(\sin A))$ if $\cos A > 0$

$A = 180° - \sin^{-1}(\sin A))$ if $\cos A < 0$

If other words, you need to know sine if you use arccosine, and you need to know cosine if you use arcsine. (Arcsine automatically gave the right result for altitude a because it's between -90° and 90°.)

2.3 The tangent point method

As shown in Figure 6, we represent circular orbits of the local standard of rest (LSR) and three hydrogen clouds (P', P, and P") around the galactic center. The sun's motion around the galactic center is not perfectly circular; the LSR is defined as a reference frame in the vicinity of the sun moving in a perfectly circular orbit. We point our telescope along a line in the galactic plane with longitude ℓ. Multiple hydrogen clouds are in our line of sight: P', P, P", and many others not shown. All of these hydrogen clouds are emitting the 21-cm line. However, the Doppler shift depends on relative speed, and we will detect a spectrum of Doppler shifts because the different hydrogen clouds have different relative speeds to us.

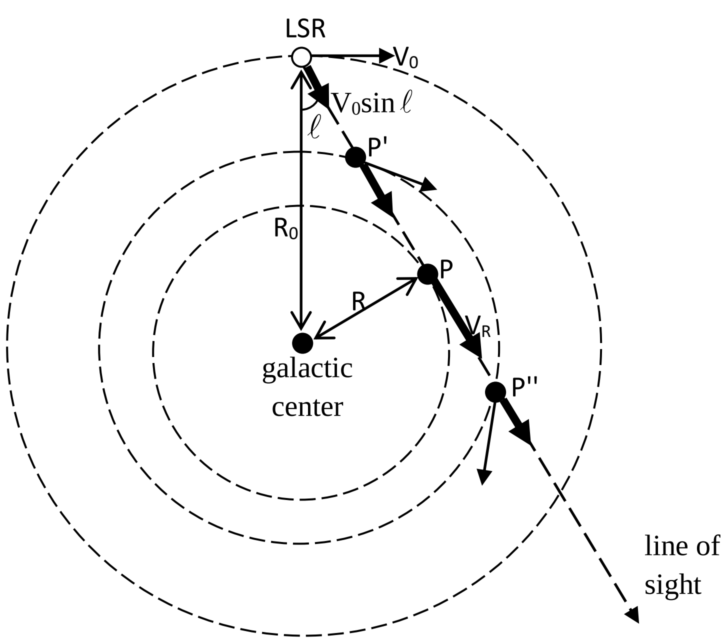

Figure 6. The local standard of rest and three hydrogen clouds (P', P, P") moving in circular orbits around the galactic center. P is the tangent point along the line of sight. The thick arrows are line-of-sight projections of velocity.

How can we unambiguously associate a specific hydrogen cloud with a specific Doppler shift in our spectrum? If we study Figure 6, we see that the greatest relative speed occurs at point P, where the entire rotational velocity is along the line of sight. (We're assuming $0° < \ell < 90°$, and that rotational speed doesn't vary too much with distance from the galactic center. If $90° < \ell < 270°$, there is no tangent point along the line of sight. If $270° < \ell < 360°$, the greatest *blue*shift occurs at the tangent point.) Since point P has the greatest relative speed to us along the line of sight, the greatest redshift that we observe will be from the hydrogen cloud at point P.

So, from our raw data for a given longitude ℓ, we identify Δf_{min}, the greatest redshift (the most negative Δf at which we observe an appreciable signal). Our ultimate goal is a plot of rotational speed V_R as a function of distance R from the galactic center. Simple trigonometry allows us to calculate R:

$$R = R_0 \sin l, \tag{2}$$

where R_0 = 8.5 kpc is the Earth's distance from the galactic center.

How do we determine V_R from Δf_{min}? The Doppler shift is

$$-c\Delta f_{min}/f_0 = v_{away} = V_R - V_0 \sin \ell + v_{LSR}, \tag{3}$$

where V_R is the rotational speed of the hydrogen cloud (away from the observer), $V_0 \sin l$ is the line-of-sight projection of the velocity of the LSR (which travels with speed V_0 = 220 km/s in a perfect circle around the galactic center), and v_{LSR} is the line-of-sight projection of the velocity of the observer relative to the LSR (due to the sun's motion relative to the LSR, the earth's motion around the sun, and the earth's rotation). To determine v_{LSR}, use http://www.gb.nrao.edu/GBT/setups/radvelcalc.html. (This website assumes the observer is in West Virginia, which is close enough to Atlanta for this purpose. The main contributions to v_{LSR} are the sun's motion relative to the LSR, and the earth's motion around the sun. These motions are independent of the observer's location on earth.)

We can solve Eq. (3) for V_R.

3. Procedure

First choose a set of coordinates in the galactic plane (b = 0). The entire fourth quadrant ($90° < \ell < 270°$) is south of the celestial equator, and much of the fourth quadrant never rises above our horizon; therefore, we should choose points in the first quadrant ($0° < \ell < 90°$). When the tangent point is near the galactic center (small R and small ℓ), the greatest redshift produces a very faint signal that is very difficult to detect. Your best results will probably be in the range $20° < \ell < 90°$, but you can try other coordinates too. I think it's fun to point the telescope directly at the galactic center. You should also take a control measurement with the telescope pointing at nothing in particular (perhaps straight up).

You will use Remote Desktop Connection to connect to the computer interfaced with the telescope. Find Remote Desktop Connection under Accessories in the local computer. Connect to the computer interfaced with the radio telescope (10.224.48.254). Please ask me for the password. Once you've connected to the remote desktop, open three applications: RT_GUI allows you to move the telescope, SEI Explorer allows you to monitor altitude and azimuth, and Spectracyber processes and records data from the radio receiver.

There are not many parameters to adjust in Spectracyber. First, choose spectral mode. You can set the range for the measurements (up to ±1000 kHz in Doppler shift), but ±500 kHz is reasonable. The other three parameters (integration time, IF gain, and DC gain) affect data quality. I seem to get the best results when I maximize integration time and IF gain, and minimize DC gain.

As the spectrum is recorded, you'll see it graphed on the screen. Be sure to export your data as a csv file to be viewed in Excel. The file name should probably indicate the galactic coordinates of your measurement.

4. Analysis

Now you have a Doppler-shift spectrum for a set of coordinates in the galactic plane. In each case, you already know ℓ, so Eq. (2) immediately gives you R of the tangent point. Now, you need to determine Δf_{min}, the greatest redshift of any appreciable signal. How do you determine Δf_{min}? One method is to simply "eyeball it" and record Δf where the spectrum just begins to rise above the baseline. (And if the baseline is slanted, which is an artifact of the electronics, just ignore the slant.) It may be helpful to plot a smoothed line.

If you want to eliminate the baseline slant, there are two methods for this. One method is to subtract your control spectrum from each of your other spectra. A possible objection to this is that the telescope may have been inadvertently pointing at a celestial radio source when you made your control measurement. To overcome this objection, you could study a catalog of celestial radio sources and make sure to point the telescope toward a quiet spot in the sky. Or, you could make control measurements of several random spots in the sky, eliminate any unusual measurements, and average the other control measurements.

A second method to eliminate the baseline slant is as follows. You can suppose that each of your recorded spectra can be represented by the function y = Signal(x) + mx + b. We want to subtract mx to eliminate the baseline slant, and we might as well subtract b to eliminate any baseline offset. To determine m and b, simply do a linear fit to the linear regions of your spectrum.

5. Lab Report

- Provide any background information required to discuss your results. It's tedious and unedifying to paraphrase this entire manual. It's more impressive if you recapitulate only the key points, perhaps clarifying and elaborating any subtleties.
- Discuss the values of v_{LSR} that you found from the online calculator. Do these values make sense, given what you know about the local standard of rest? How would v_{LSR} differ if you repeated the experiment in six months? Why?

- Present your rotation curve. What does it imply about dark matter, given that the luminosity of the galaxy is concentrated in the center? What would the rotation curve look like if we assumed that outer masses in the galaxy orbited a much larger, central mass?