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Superfluid Drops: Dynamics of Pinch-Off and Sliding Motion

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We present high resolution and high speed (5 μs) photographs of ^4He drops undergoing pinch-off and sliding down a cesiated inclined plane. When a fluid drop is stretched and pulled apart by gravity, a balance of surface tension and inertia results in a striking icicle-shaped column of fluid which connects the two separating parts. The narrowest point of the icicle is an example of a finite-time singularity in the equations of motion. The tip radius of the icicle L obeys a power law $L \sim \tau^{2/3}$, where τ is the time before the moment of pinch-off. We have verified this for both superfluid and normal drops. Because of the boundary condition requiring zero velocity at a solid wall, sliding and rolling motion of drops on a substrate is a subtle issue even for conventional fluids. For example, calculations of the dissipation yields nonphysical infinities. We have analyzed video images of sliding superfluid drop motion and measured the acceleration of ^4He droplets on a Cesium substrate.

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1. INTRODUCTION

The purity and superfluidity of liquid ^{4}He make it an excellent system for studying classical fluid dynamics problems. In this paper, we discuss the uses of ^{4}He in investigations of pinch-off which occurs when two fluid masses separate, and sliding motion of droplets down a non-wetted surface. The equations describing droplet pinch-off are strongly non-linear and exhibit a finite time singularity; similar equations have been applied to other topology-changing transitions in fields ranging from cosmology to biology. Pinch-off behavior is dominated by surface tension and density in ^{4}He, and viscosity is only a perturbation. In the pinch-off studies, our main goal was to explore

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the possible effects of superfluidity. In contrast, superfluidity is expected to have a large effect on sliding droplet motion. An analysis of this problem for ordinary viscous fluids using the Navier-Stokes equations with the conventional no-slip boundary conditions gives infinities in the dissipation[1]. This implies that the no-slip boundary condition must break-down near the contact line. Superfluid drops provide an opportunity to decouple the effects of viscosity from dissipation mechanisms that are intrinsic to contact line motion.

2. EXPERIMENT

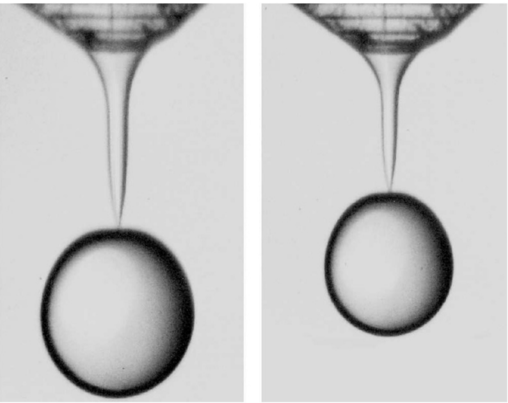

Fig. 1. Superfluid(left, 1.4K) and normal fluid(right, 2.9K) droplets at the moment of pinch-off. Although the drops differ in size due to the temperature dependence of the surface tension, the pinch region is similar in both. The tick marks on the ruler (top of picture) where the drops begin are .1 mm apart.

We have built an optical cryostat that employs evaporative cooling of ^{4}He to maintain temperatures in the experimental cell as low as 1.4K. Most previous[2,3] investigations of pinch-off have used high viscosity fluids in which the pinching time scale is of the order of seconds. For low viscosity fluids, the process happens in a few milliseconds, so we utilized a laser-triggered xenon flash tube to capture the images on a digital still camera. A delayed-pulse generator was used to control the time between the trigger and the flash.

For sliding motion of droplets, we used a 1000 frame-per-second video camera to measure the drop position in sequential frames. From these images, the drop's position, velocity, and acceleration can be determined. The drops were rolled down a polished silicon substrate on which approximately 100 layers of cesium were deposited. The surface was dried off before each drip using a small heater sputtered onto the underside of the silicon.

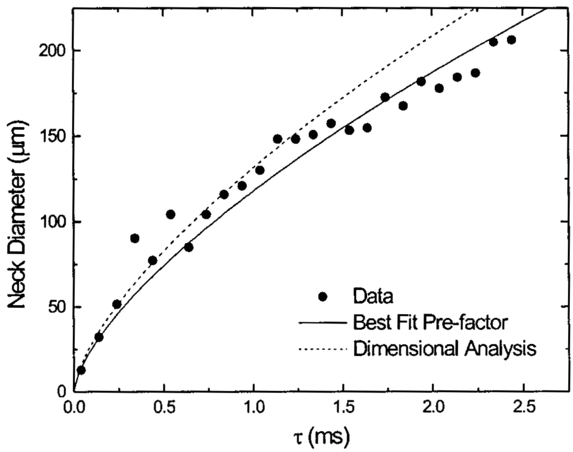

Fig. 2. Superfluid (T=1.4K) neck diameter vs. time from break. The solid line assumes a 2/3 power-law behavior, and the pre-factor is fitted to the data. The dashed lines are taken from Eq. 1, with no pre-factor multiplication.

3. RESULTS AND DISCUSSION

A relationship between the characteristic length and time scales in the problem can be derived from the nonlinear equations of potential flow (neglecting viscosity) using dimensional analysis. Assuming that inertia and surface tension are the two driving forces, one obtains :

$$L = \left(\frac{\sigma\tau^2}{\rho}\right)^{1/3} \tag{1}$$

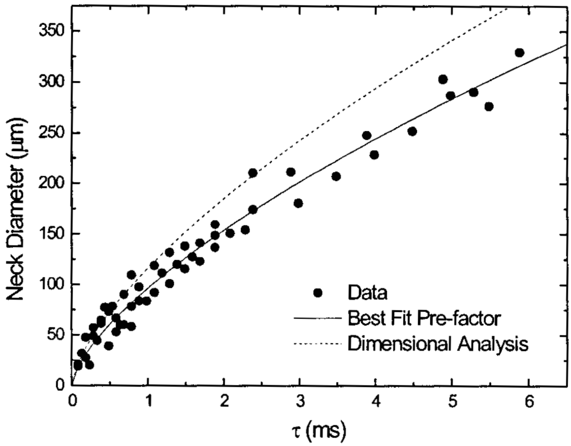

Fig. 3. Normal fluid (T=2.9K) neck diameter vs. time from break. The solid line assumes a 2/3 power law behavior, and the pre-factor is fitted to the data. The dashed lines are taken from Eq. 1, with no pre-factor multiplication.

Where L is the characteristic length taken as the neck diameter at the pinch, and τ is the time from the break. The parameters σ and ρ represent the liquid-vapor surface tension and density of the bulk fluid, respectively. Another dimensional argument suggests that the effects of viscosity should become dominant at times $\tau \approx 10^{-11}$ seconds. Figure 1 shows pictures of normal and superfluid at the moment of pinch-off; Figures 2 and 3 show plots of the minimum neck diameter as a function of time from the singularity at τ=0. Both normal and superfluid ^{4}He appear to follow the 2/3 power-law behavior predicted by dimensional arguments. The difference in size of the two drops is due to the variation of the capillary length as a function of temperature. The density and surface tension of ^{4}He do not change dramatically through the λ-point, so the physics of the problem is essentially the same above and below the superfluid transition. Our optical system has a resolution of about 10 μm, which limits our observations to $\tau > 10^{-4}$ sec. To follow the singularity to substantially smaller length scales, a non-optical probe is required. We have developed a new technique to measure pinch-off by monitoring the resistance of a conductive fluid such as mercury. This method allows us to

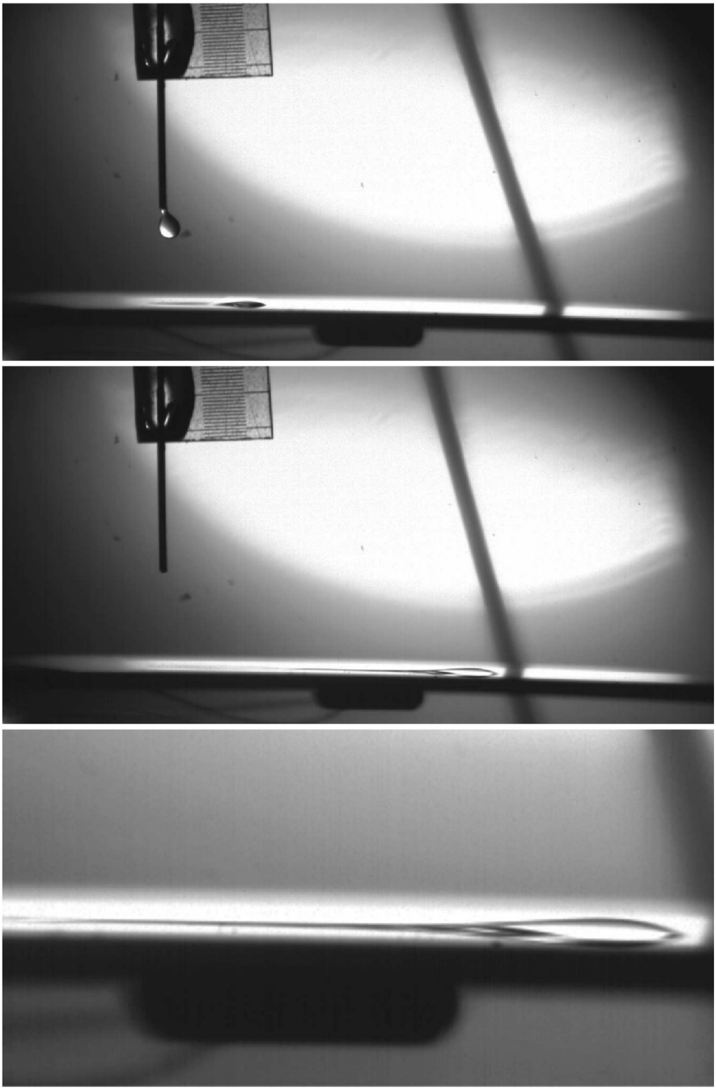

Fig. 4. Sample frames from the high-speed video camera at T=1.53K and θ=17 degree incline. The first frame shows the capillary delivering drops to the substrate. The frame show the drop sliding down the incline, and the last frame is a magnified picture of the sliding droplet where the tail can be seen. The large tick marks on the ruler attached to the capillary are 1 mm apart, and the black tilted line on the right is a plumb bob which helps to define the gravitational vertical.

track the time dependence of the neck diameter down to 30 nm.

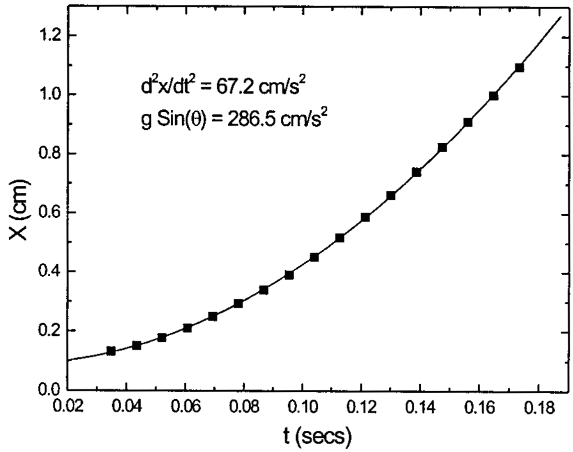

Fig. 5. Position vs. time graph for droplet at T=1.53K and θ=17 degree incline. It has an excellent quadratic fit, indicating a constant acceleration of 67.2 cm/s^2, much smaller than gravitational acceleration alone.

Our interest in the motion of superfluid drops stems from our previous work on the properties of static He drops[4]. By analogy to the elementary physics problem of a block on an inclined plane, our previous experiments showed that the effective static friction coefficient of helium on Cs was high. Our current experiments are designed to measure the coefficient of sliding friction of superfluid ^{4}He droplets on a cesiated inclined plane. The 1000 frames per second camera allowed us to capture the entire motion of the slide with 10 μm accuracy. A sample of frames can be seen in Figure 4. Since both the camera and the cesiated substrate are tilted, the substrate appears horizontal in the picture; the angle of incline can be determined from a plumb line that marks true vertical. We obtained position vs. time graphs for differing inclines and temperatures by tracking the advancing edge of the drop in sequential frames (Figure 5). The data shows a constant acceleration for the droplets, with no evidence for a terminal velocity. However, the acceleration is much smaller than what would be produced by gravity alone, which implies the existence of a frictional force. One possible mechanism for a constant drag force is the surface tension of the long trail of fluid that is pulled behind the droplet (Figure 4). This additional surface area costs a considerable amount of energy, which reduces the overall acceleration. Assuming the mass lost to the tail is negligible, the acceleration can be expressed as:

$$a = g\sin(\alpha) - \frac{\sigma_{lv} d(1 - \cos(\theta))}{m} \tag{2}$$

The parameters include α (angle of incline), σ_{lv} (the liquid-vapor surface tension), d (the width of the liquid trail), θ (the advancing contact angle), with g and m being gravitational acceleration and the mass of the drop, respectively. We are currently investigating the effects of drop mass and temperature implied by Eq. 2. We also plan to vary the properties of the Cs substrate. Previous[5] experiments have shown that annealing the substrate at 77K and lowering the temperature to $\leq$ 1.2K suppressed the formation of the tail, and created a receding contact angle of about 7 degrees. If the mechanism described by Eq. 2 is valid, drops on this type of substrate should have a distinctly different acceleration.

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