INT. J. MATH. EDUC. SCI. TECHNOL., 2004
VOL. 35, NO. 5, 669–679

Taylor & Francis
Taylor & Francis Group

# Cooperative study teams in mathematics classrooms

CHARALAMPOS TOUMASIS
33 Norman Street, 26223 Patras, Greece
E-mail: chartoum39@hotmail.com, toumasis@sch.gr

(*Received 26 September 2003*)

This article describes a general instructional strategy designed to help students in the learning process from textbooks and to furnish opportunities for practice in critical reading. Students participate in cooperative learning by breaking the class up into small groups—the Study Teams—and providing them with worksheets and reading organizers, which organize the material into small items that reflect the major concepts in the reading material on which the study is focused. Some of the benefits that this type of instruction with Study Teams can produce are described.

## 1. Introduction

As mathematics teachers, we often voice complaints about our students' reluctance to read their mathematics textbooks, or their difficulties solving word problems due to poor reading skills. The typical mathematics student expects a teacher to explain what the book says. He may read a chapter in a history text and answer questions in the book without asking for any explanation; but in mathematics, after the student reads the material the typical response is: 'Fine, I read it. But what does it mean?'

Mathematics is a language that can neither be read nor understood without initiation. Students need to learn how to read mathematics, in the same way they learn how to read a novel or a poem, listen to music, or view a painting. Both a mathematics article and a novel are telling a story and developing complex ideas. The greatest difference is that a mathematical article does the job with a tiny fraction of the words and symbols used in a novel. In reading mathematics each word or symbol is important because there are many thoughts condensed into a few statements.

Mathematics is one area of the curriculum where traditionally, little reading occurs. For a variety of reasons, students do not know how to study mathematics [1]. Most of the time spent deliberately helping students learn to read focuses on literary and historical texts. Mathematical reading (and for that matter, mathematical writing) is rarely expected, much less considered to be an important skill, or one which can be increased by practice and training.

However, in a period of rapid technological change this situation is distressing and dangerous as well. The author believes that the school of the future will have to concentrate more on the relationship between people and knowledge than on knowledge itself. Increased use of information technology has led to an increase in the incidence with which quantitative information is presented in the printed

*International Journal of Mathematical Education in Science and Technology*
ISSN 0020–739X print/ISSN 1464–5211 online © 2004 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/0020739042000232529

media. There is a perceived need for citizens to be able to interpret such information. The pressure for this has been felt more and more in mathematics and science [2, 3].

In a time where there is a need for frequent adaptation of skills, it is generally agreed that teachers must do more than just teach students a certain body of facts. They are responsible for teaching students processes of thinking and learning so that they have the ability to become increasingly self-directed and not depend on the teacher [4]. In spite of advances in the electronic media the book still remains the best place to find and study a very large body of information. Current software can advance the printed media, but not replace it.

Therefore, the ability for critical reading may well be more valuable to students in the future than the actual topics that they study in the course. NCTM's Curriculum and Evaluation Standards for School Mathematics [5], emphasizes the importance of reading skills for all students and points out the fact that they will need specific instruction on how to read mathematical textbooks with understanding. 'Assignments that require students to read mathematics and respond both orally and in writing to questions based on their reading should be an integral part of the 9–12 mathematics program' (p. 142).

Cooperative learning has also been a subject of interest to researchers for the past three decades, and many research findings indicate that cooperative learning is an effective tool for improving academic achievement [6–9]. At the K-12 school levels, instruction using cooperative or collaborative learning techniques has gained in popularity, and there is a substantial body of literature supporting the idea that students can attain higher achievement, especially in mathematics, through working together in groups [10–14].

Until now, however, there has been no specific effort to focus on the importance of cooperative techniques designed to help students in reading mathematical textbooks with understanding. A goal that should be a conscious part of our teaching efforts is to teach our students to read and understand mathematics. This paper advocates this point of view. It presents a general instructional strategy designed to help students in the learning process from textbooks and to furnish opportunities for practice in critical reading. The author has used this technique during regular classroom instruction with first- and second-year algebra students, and all experiences described occurred in these teaching sections.

## 2. Instruction with cooperative Study Teams

The main idea of this strategy is that by participating in cooperative learning, students are able to learn concepts, processes and techniques presented in the textbook while working under the supervision of the classroom teacher. The following instructional sequence can be implemented:

1. Teacher introduces the unit to the class.

The unit may be a chapter section in an elementary mathematics textbook that can be taught during one class period: for instance, the quadratic equation and the nature of its roots. The teacher offers a unit overview in the whole class, which may include a general discussion about the unit objectives, a film or other media to generate excitement about the topic.

2. The whole class is divided into Study Teams.

The class is broken into groups of four or five students who constitute the Study Teams. The members of each group work together at one time following written instructions and are responsible for cooperating on learning tasks using the mathematics textbook.

Each Study Team must be a heterogeneous or mixed ability group. With a little planning, it is possible to have high, average and low-achieving students in each Study Team. In this way it is possible to pair a marginal student with an average or above average student for peer instruction and support, which will assist the marginal learners in improving their learning and self-confidence. Each student can be given a colour team badge to enhance the collaboration spirit of each team. Collaborative learning in the mathematics classrooms is a teaching strategy that holds increased promise for improving the mathematical skills and attitudes of students [15]. 'Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing' [5, p. 79]. Each team also selects a recorder-reporter who is responsible for reports to the whole class without mentioning any individual names in the report.

3. Teacher distributes Study Team worksheets.

Each worksheet contains a list of reading organizers that tells the learners what is important in each paragraph of the unit. The items on each sheet are questions based on the main ideas, concepts, processes and generalizations found in the unit. These items reflect the major concepts in the reading material on which the study must be focused. There is approximately one reading organizer for each paragraph. Appendix A presents a usable Study Team worksheet developed from an algebra textbook chapter on quadratic equations. If a chapter divides into five units, for instance, the teacher must prepare five worksheets and there will be five Study Team sessions.

4. Students learn concepts identified on the worksheet cooperatively.

In this session, students use the reading organizers to read paragraphs, point out portions, discuss concepts, keep notes on difficult areas, and agree on responses. Students read the textbook reflectively using pencil and paper. Rather than just keep on reading and waiting to see what the author's explanation is, they try to transform the content in such a way as to produce understanding. The advantage is that group therapy and peer learning is achieved. Students feel secure in the Study Team. Once the group has started its work then each member knows that the group will assist if he or she gets stuck or has been absent and needs information or did not understand something. Students can relate to one another better than to an instructor, at least when none of them understands a concept. It is fascinating to see the effects of this approach on students. Once when the author stopped to ask a member of a team if he needed help, he said: 'Maria is helping me!' giving me a funny look as if to say 'Go away'.

During the Study Team session the teacher moves among the groups organizing the class work, answering students' questions, explaining what the symbols mean, drawing out their ideas, giving suitable prompts and encouragement and facilitating their work. The instructor feels he is working 'with' students instead of 'on' them. The responsibility for working and learning becomes the students',

not the teacher's. The students learn that a teacher helps to guide, but cannot learn for them.

5. Class discussion.

After the collaborative reading and learning guided by the worksheet the teacher allows time for the whole class to discuss those areas that need clarification involving students in a whole class discussion. The recorder from each team selects the difficult topics that the members have pointed out in their notebooks and asks the teacher to clarify them. The discussion session helps the class to elucidate key ideas and explain difficult points eliminating the possibility that an incorrect response has been given in the Study Teams. At this time, the teacher can correct errors, expand concepts and give further explanatory information or short demonstrations. At this session the teacher can also use supplementary materials that may include diagrams, graphs, or restudy on the unit topic (figure 1).

After this discussion each member of the Study Team completes its worksheet. A grade can be given to each student for completing the worksheet.

6. Teacher distributes summary cards.

If students have really understood something they are expected to explain it, and furthermore, requiring them to explain it helps them to understand the information. Verbalizing and writing also improves the ability to recall and organize information and serves as a powerful aid to learning [16].

Each member of the Study Team is given a $3 \times 5$ card and the students are asked to write on it, in their own words, a brief summary of the main points of the lesson. The students are encouraged to look at their textbooks and answer the open-ended request: 'What did I learn today?' (figure 2). This technique of writing on mathematics-related content helps students clarify and develop their understanding of a specific topic, as when putting down ideas on paper, they are faced with them and obliged to recognize their shortcomings. A grade can be given to each student for completing the summary card.

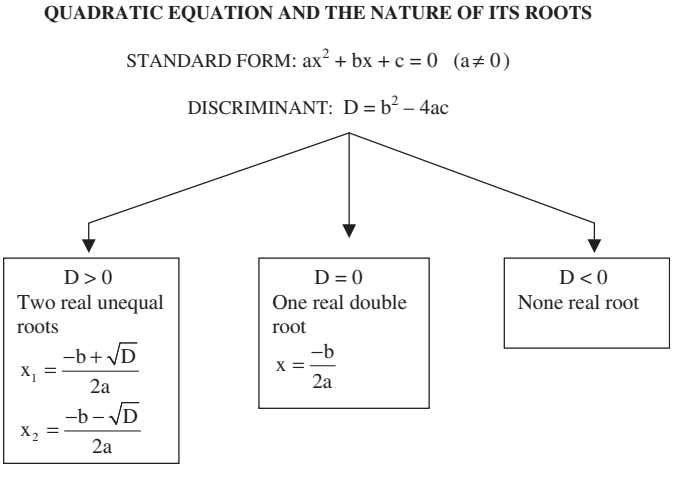


Figure 1. Conceptual diagram of the study unit.

Today I learn that:

The standard form of a quadratic equation is:
$ax^2 + bx + c = 0 \quad (a \neq 0)$.
The discriminant of a quadratic equation is: $D = b^2 - 4ac$.
The quadratic formula is $x = \dfrac{-b \pm \sqrt{D}}{2a}$.
The discrimination sign determines the nature of roots.

Figure 2. A student's summary card.

7. Students take individual tests.

Up to this point, the Study Team members have received identical grades for the group work on worksheets or the summary cards. The individual unit test is used to assess student mastery of the concepts. The test items should be based on the reading organizer worksheets and include all facts, concepts, and skills that are based on the unit as taught. This alignment between the unit as taught and the evaluation of students' outcome is important for slow learners and low ability students. Appendix B presents a list of focused study items and the corresponding evaluation items. Finally, each student receives a unit score based on a weighed combination of the collaborative group work and the individual test score.

## 3. Methodology

This article is a report on the experimental use of cooperative Study Teams in four secondary mathematics classrooms. The experiment has been running for three years (1999–2002), is integrated in an existing mathematics curriculum, and has included approximately 100 students in grade levels 8–10.

The experiment took place at three urban secondary schools in Patras, Greece. The student population was drawn from diverse socio-economic backgrounds and approximately half of each class were girls and half were boys. Cooperative learning skills such as positive interdependence, roles in a group, rules and procedures were all reviewed before the lessons began. The study was conducted two times a week during a 4-week period each time and covered a specific mathematics unit.

Finally, the participants were asked to reflect in writing, on the totality of their experiences at the conclusions of the teaching experiment. Students were asked to reflect on statements about mathematics and they were challenged to articulate their own perceptions of mathematics and their experiences with cooperative Study Teams.

These reflections, and also their reactions to the seven instruction sessions, were recorded in a class journal which circulated among the participants at the end of all study items and the corresponding evaluation items. The journal was returned to the author after two weeks. Since journal writing was new to the students, it was discussed with them, focusing on the purpose of keeping a journal and guiding them in the use of this activity [17].

In order to help them to express their feelings, it was proposed that they reflect on five items:

(1) How has cooperative Study Teams affected your learning of mathematics?
(2) How do you feel about instruction with cooperative Study Teams in math classes?
(3) What are the benefits of cooperative Study Teams in math classes?
(4) How could this instruction strategy be changed to be more effective?
(5) Would you like to continue this activity in mathematics?

Overall, the comments were very favourable and encouraging. Representative student comments included:

> 'It was great to get into mathematics a little. It broadened my horizons'.
>
> 'I learned to value cooperative work. It was fun and made very interesting by the instructor'.
>
> 'Working in a group can sometimes be a challenge because it helps in putting different ideas into perspective and allows verbalization of results, which is very helpful'.
>
> 'This has been one of the most beneficial courses I have taken'.
>
> 'I learned to use the text as the source of facts and not to depend on the teacher'.
>
> 'The more work that we do with quadratics and factorization, the more interesting it becomes. I never looked at math in such a way as we do now'.
>
> 'I liked today's class. I can't believe we came up with a theorem all on our own'.
>
> '... It's too bad the experiment period is almost over'.
>
> 'I felt ... some sense of satisfactory and mastery. With help from another classmate I solved the problem at the end of the session'.
>
> 'You are broadened by new ideas, somebody else's viewpoint. You're able to see more'.
>
> 'For a moment we tried to solve the problem but we didn't and then we read. But our mistake was that we didn't take a pencil and a piece of paper and draw something'.

In addition, some weak students were interviewed before and after the sessions to determine their background and past experience with mathematics, and to allow them to express their feelings about mathematics.

## 4. Concluding remarks

The conclusions and comments expressed in the following come from reflecting on the author's teaching experience and in an analysis of the comments found in the journal writing of the many students who form the subject of the study. This evidence shows the possibility of engaging students in small-group

mathematical activities with the purpose of helping them in the learning process from textbooks on mathematics-related content.

Certainly, we have not done a comparative analysis of these classes to see if this teaching strategy makes a difference in how much students learn, but we are convinced it helps. Without having provided conclusive evidence, we would like at least to suggest the hypothesis that involving students in different study experiences could contribute in a complementary way to their learning of mathematics.

Using this general instructional sequence, a single chapter can be taught in five to eight instructional periods. However, many extensive activities can be inserted between the several steps or some shortened or omitted. This depends on the ability level of the class and the background of the members in each group. We have found that the Study Team technique for learning from textbooks is easily adaptable to all secondary mathematics classes whatever the grade level but works best in mixed ability classes with a great number of low achievers and anxious students.

The use of this type of instruction with Study Teams has several advantages.

(1) It can help motivate students because everyone is involved in discussing and learning the material.
(2) Students are motivated to participate in meaningful reading and language experiences, to identify important concepts and to think about the meaning of these concepts.
(3) Study Team membership and peer tutoring give help to slow or disinterested students with poor mathematical backgrounds, bad experiences in mathematics, and bad attitudes who seldom receive classroom recognition. Students who know that they can depend on other group members to help and support do not feel the anxiety often experienced by those who do not understand the work. The classroom environment is less threatening, and anxiety is less likely to interfere with learning. Moreover, the student who helps others experiences gratification in giving.
(4) Students through Study Teams form new friendships and learn to appreciate differences in ability, differences in personal characteristics and differences in opinion. The cooperative-learning attitude offers a secure environment for everyone to make a contribution. Each student feels responsible for his/her own team performance and is rewarded for his/her contribution.
(5) Students are taught to read mathematics textbooks critically with systematic note-taking, outlining the most important areas, making connections between pictures, examples and diagrams, using pencil and paper and trying to interpret what the author is describing with symbols and words. This reading activity draws learners into the texts and encourages them to raise questions, make connections and, in general, actively work out meanings with the support of peers.
(6) Students also learn to use the text or their own summary as the source of facts and not to depend on the teacher for this information. They are expected to develop good study habits at their own level and prepared for future self-study. The mere fact that class time is devoted to reading

the textbook demonstrates to the students that the teacher values reading as a learning activity.

(7) Their teacher is no longer seen as the authority that dispenses knowledge to students, who merely absorb information. Instead, students become important resources for one another in the learning process.

During the last three years since the author has used this technique in the traditional classroom setting, most of the students have shown enjoyment in the explanatory nature of the learning process and appreciated the opportunity to work together rather than in silence on their own as in the traditional courses. Students' attitude, attendance, completion of assignments and willingness to participate in class has improved. In particular, the so-called 'poor' and unmotivated students have shown extraordinary creativity.

However, in the author's opinion, it is inappropriate to discuss or try to assess and compare different teaching methods or styles. Instead it should be more fruitful to ask whether the method chosen works or not with that particular instructor and that particular class and whether the method led to the goal set by the particular teaching sequence. With regard to our case, it seems that the Study Teams activities work well, that students learn their mathematics as well as developing good habits for future action, and that they like the method. Most of them are enthusiastic about trying something new and make an honest attempt to carry out the assignments.

Other colleagues who may want to use similar techniques based on the principles discussed in this article must have in mind that in order to operate in this way many students have to 'unlearn' habits built up over many years at school. Students may come from schools or classes with preconceptions about how they should behave in a mathematics class. The spirit of cooperation may be alien to students who have been schooled in an environment where the teacher carries the entire burden of the instruction and the students compete with one another to rise to the top. Therefore, it is recommended that these ideas should be continued for at least five courses before the students begin to accept them. This approach seems to have great promise and, with minor revisions each time, may challenge the near monopoly of the lecture as a principal means of mathematics teaching.

## Appendix A Sample Study Team worksheet

### READING GUIDE

DIRECTIONS: Meet with your team to read from your textbook about quadratic equations and the nature of their roots.
Use pencil and paper for note taking. Then discuss and write down the responses to the following items.

QUADRATIC EQUATIONS AND THE NATURE OF THEIR ROOTS
(Pages_____)

1. Is there a standard form for a quadratic equation? _____
   If so, what is it? _____

2. Describe the process of completing the square.

   _____

   What is its purpose? _____

   _____

3. In terms of a, b and c, what is the discriminant, D?

   _____

   _____

   What is the significance of the discriminant? _____

   _____

   _____

4. Write the quadratic formula: x =
   What is its use? _____

5. Do all quadratic equations have two roots? _____
   Explain why or why not. _____

   _____

6. When are a quadratic equation's roots rational and when are they irrational?

   _____

   _____

7. Upon what does the nature of the roots depend? _____

   _____

8. What is implied when D > 0? _____
   When D = 0? _____
   When D < 0? _____

## Appendix B Focused study items and corresponding test items.

QUADRATIC EQUATIONS AND THE NATURE OF THEIR ROOTS

| FOCUSED STUDY ITEMS | TEST ITEMS |
|---|---|
| The standard form of a quadratic equation is (page . . . . . ) . . . . . | What are the a, b,c for the following quadratics: i) $3x^2 - x + 2$ ii) $x^2 - 7$ iii) $x^2 - 3x$ . . . . . |
| The discriminant of a quadratic is (page . . . ) . . . . . | Compute the discriminant of the above quadratics . . . . . |

The quadratic formula is (page . . . . . )
. . . . . .

a) Find the roots of the equations

i) $2x^2 - x - 6 = 0$
. . . . . .
. . . . . .

ii) $t(3t - 10) = 25$
. . . . . .
. . . . . .

b) The formula $K = n(n - 3)/2$ yields the number of diagonals, K, in a polygon of n sides. Find the number of sides of a polygon having 54 diagonals.
. . . . . .
. . . . . .

The nature of roots depends on (page . . . )
. . . . . .
. . . . . .

a) Find the value of the discriminant for each of the following quadratic equations and use it to tell how many real roots the equation has.

| | D | N° of real roots |
|---|---|---|
| $x^2 - 3x + 2 = 0$ | . . . . . | . . . . . |
| $x^2 - 5x + 4 = 0$ | . . . . . | . . . . . |
| $x^2 - 7x + 6 = 0$ | . . . . . | . . . . . |
| $x^2 - 9x + 8 = 0$ | . . . . . | . . . . . |

Can you find the pattern? . . . . . .
. . . . . .
Explain, by the means of D, why this form of quadratic equations have always real roots
. . . . . .
. . . . . .
Try to find another general form, which would give you quadratic equations which have always real roots . . . . . .
. . . . . .

b) Repeat the same work for the following equations:

| | D | N° of real roots |
|---|---|---|
| $2x^2 - 2x + 1 = 0$ | . . . . . | . . . . . |
| $5x^2 - 4x + 1 = 0$ | . . . . . | . . . . . |
| $10x^2 - 6x + 1 = 0$ | . . . . . | . . . . . |
| $17x^2 - 8x + 1 = 0$ | . . . . . | . . . . . |

Can you find the pattern? . . . . . .
. . . . . .

Explain, by the means of D, why this form of quadratic equations have always no real roots.
. . . . . .
. . . . . .

Try to find another general form, which would give you quadratic equations which have always no real roots . . . . . .
. . . . . .
. . . . . .

## References


[1] MARGENAU, J., and SENTLOWITZ, M., 1977, *How to Study Mathematics* (Reston, VA: National Council of Teachers of Mathematics).

[2] NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 2000, *Principles and Standards for School Mathematics* (Reston, VA: The Council).

[3] NATIONAL COMMISSION ON MATHEMATICS AND SCIENCE TEACHING FOR THE 21ST CENTURY, 2000, *Before it's Too Late: A Report to the Nation* (Washington, DC: US Department of Education).

[4] MOSES, R. P., 2001, *Radical Equations: Math Literacy and Civil Rights* (Boston, MA: Beacon Press).

[5] NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 1989, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: The Council).

[6] JOHNSON, D., and JOHNSON, R., 1987, *Learning Together and Alone: Cooperation, Competition, and Individualization*, 2nd edn (Englewood Cliffs, NJ: Prentice-Hall).

[7] SLAVIN, R., 1983, *Cooperative Learning* (New York: Longman).

[8] SLAVIN, R., 1987, *Child Development*, **58,** 1161.

[9] WEBB, N., 1985, Student interaction and learning in small groups: A research summary. *Learning to Cooperate, Cooperating to Learn*, R. Slavin, S. Sharan, S. Kagan, R. Hertz-Lazarowitz, and C.Webb (eds), (New York: Plenum), pp. 147–172.

[10] DAVIDSON, N., 1990, Small-group cooperative learning in mathematics. *Teaching and Learning Mathematics, 1990, Yearbook of the National Council of Teachers of Mathematics,*T. J. Cooney and C. R. Hirsh (eds), (Reston, VA: NCTM), pp. 52–61.

[11] LEIKIN, R., and ZASLAVSKY, O., 1997, *J. Res. Math. Educ.*, **28,** 331.

[12] LEIKIN, R., and ZASLAVSKY, O., 1999, *Math. Teacher*, **92,** 240.

[13] SUTTON, G. O., 1992, *Math. Teacher*, **85,** 63.

[14] WEBB, N., TROPER, J., and FALL, R. 1995, *J. Educ. Psychol.*, **87,** 406.

[15] DAVIDSON, N. (ed.), 1990, *Cooperative Learning in Mathematics: A Handbook for Teachers* (Menlo Park: Addison-Wesley).

[16] NAHRGANG, C., and PETERSON, B., 1986, *Math. Teacher*, **79,** 461.

[17] BORASI, R., and ROSE, B., 1989, *Educ. Studies Math.*, **20,** 247.