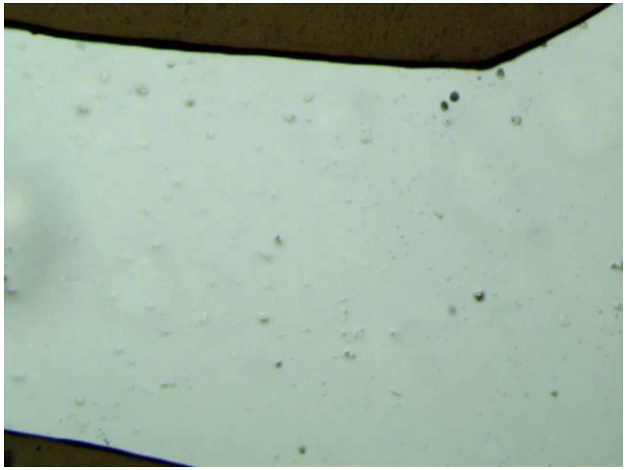
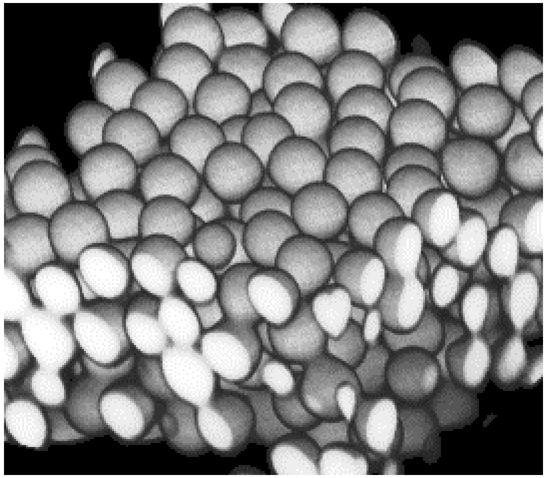
Introduction to soft materials

Eric R. Weeks
Physics Dept., Emory University
Atlanta, USA

With additional assistance from
Xin Du 杜鑫
Xia Hong 洪霞



above: emulsion

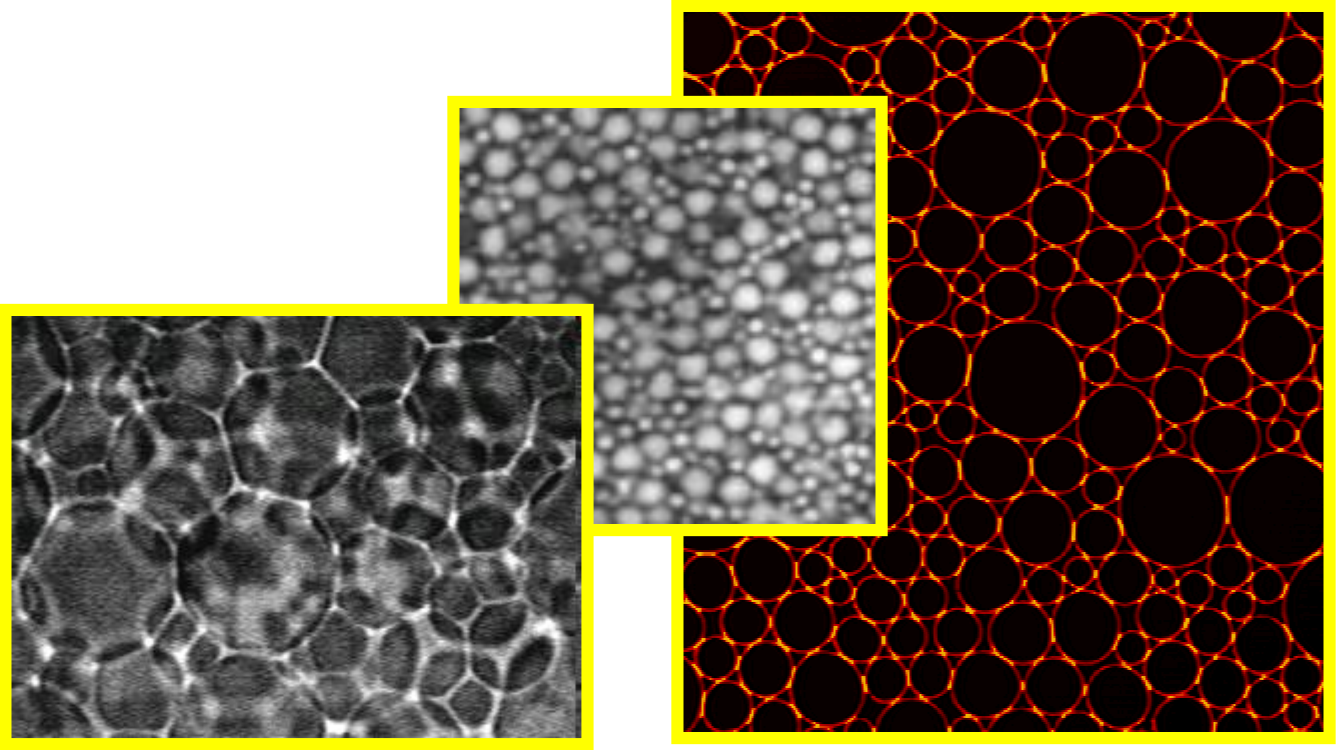


above: colloid

toothpaste (colloid) and shaving cream (foam)

Today’s class:

- Introduction to specific soft materials (colloids, foams, emulsions…)
- Relevant physics for thinking about these materials



Examples of soft materials

- food
- toothpaste
- shampoo
- sand piles
- foam

Key theme: often mixtures of various components

Photo: Piotr Habdas

Why study soft materials?

- Lots of cool physics! (some to be discussed in this talk)
- Also some practical reasons…

Why study soft materials?

- Food! (pictures from a cafeteria in Shanghai)

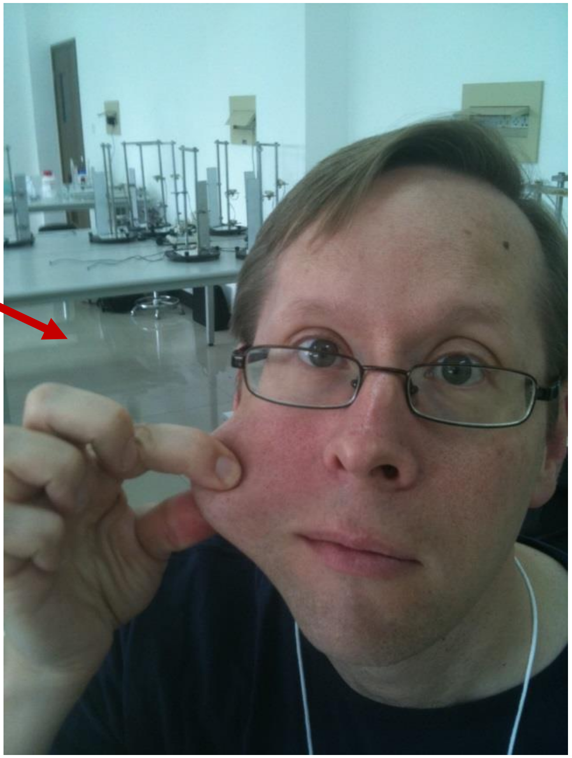
rice noodles (粉丝)

sponge gourd + gluten (丝瓜面筋)

- Make low-fat products with same texture as "normal"
- Improve "shelf-life": food texture changes with time

Why study soft materials?

- Biology: Understand mechanical properties of cells, tissues



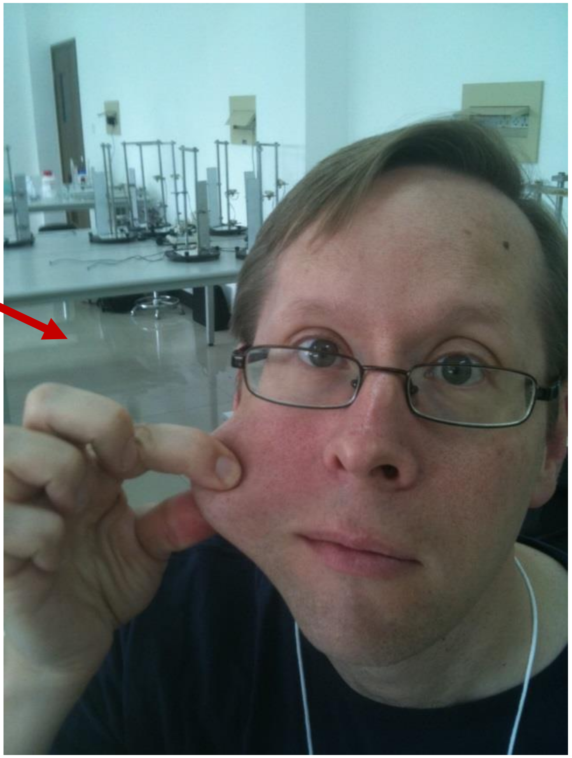
people are soft materials

photo credit: Xin Du

Why study soft materials?

- Biology: Understand mechanical properties of cells, tissues

people are soft materials



- Physics: plenty of interesting physics to do!

photo credit: Xin Du

soft condensed matter

soft condensed matter

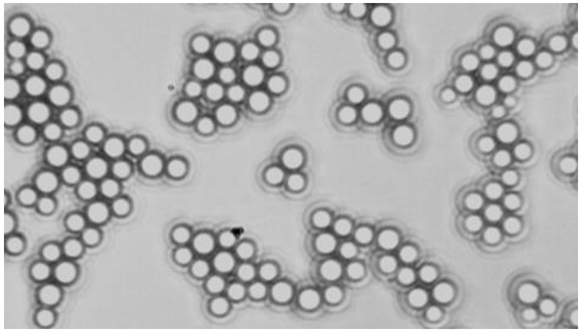
Study of soft systems, often composed of at least two components. Examples: foams, emulsions, colloids, polymers, gels, pastes, food, … Sometimes called “complex fluids”.

Key question: how to relate microscopic structure to macroscopic properties.

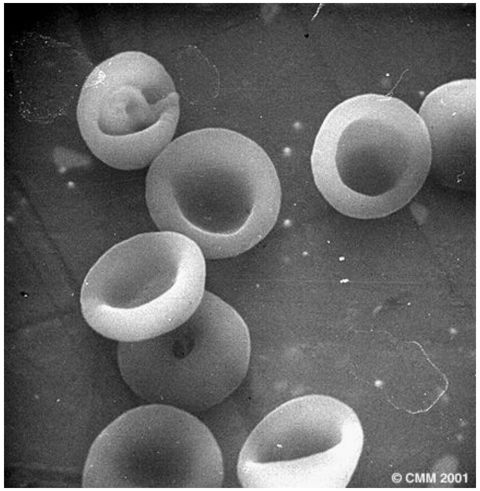
Example A: Colloids

Examples: milk, ink, paint, toothpaste, blood

- 1 nm - 10 μm solid particles in a liquid
- $k_B T$ important
- study with visible light ~ 0.5 μm
  (microscopy, light scattering)
- reasonable time scales



1-2 μm dia. colloids
(E. Weeks & H. Patel)



© CMM 2001

Red blood cells (5 μm dia.)
(http://www.uq.edu.au/nanoworld/)

Physics of colloids: Brownian motion

Leads to normal diffusion:

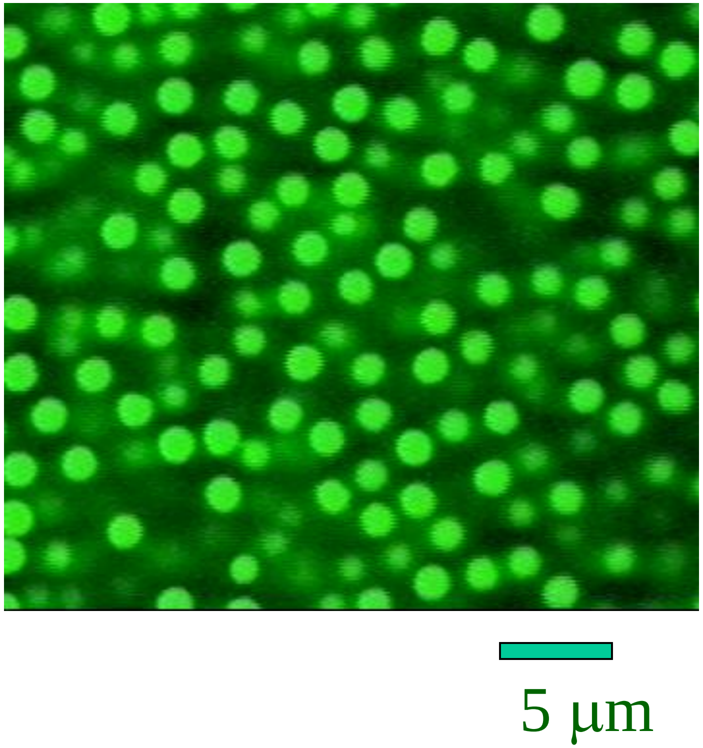
$$\langle \Delta r^2 \rangle = 6D\Delta t$$

$$D = \frac{k_B T}{6\pi\eta a}$$

viscosity η

particle radius *a*

a = 1 μm particles



Digression:
Stokes-Einstein-Sutherland equation

$$\boxed{D = \frac{k_B T}{6\pi\eta a}}$$

Derived in 1905:

W. Sutherland, *Phil. Mag.*, **9**. 781.

A. Einstein, *Ann. der Physik*, **17**, 549.

Willian Sutherland in his twentieth year.

www.aapps.org/archive/bulletin/vol15/15_1/15_1_p35p36.pdf
antwrp.gsfc.nasa.gov/ apod/ap000108.html

Digression: Stokes-Einstein-Sutherland equation

$$\boxed{D = \frac{k_B T}{6\pi\eta a}}$$

On the motion of small particles suspended in liquids at rest required by the molecular-kinetic theory of heat
Einstein, *Annalen der Physik*, 17(1905), pp. 549-560.

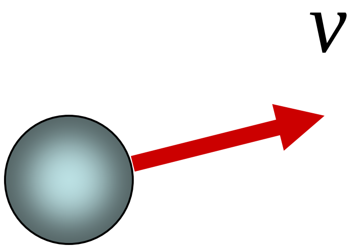
Implication: Avogadro’s number

A dynamical theory of diffusion for non-electrolytes and the molecular mass of albumin
Sutherland, *Philosophical Magazine*, S.6, 9 (1905), 781-785.

Implication: size of albumin

See: www.aapps.org/archive/ bulletin/vol15/15_1/15_1_p35p36.pdf

Physics of colloids: Sedimentation



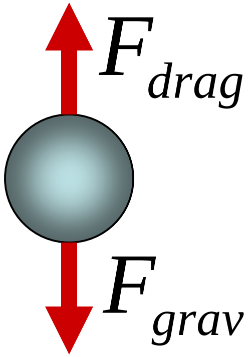
Stokes drag force: $F_{drag} = 6\pi\eta a v$

viscosity η

particle radius *a*

Gravitational force: $F_{grav} = mg \rightarrow \left(\tfrac{4}{3}\pi a^3\right)\left(\Delta\rho\right) g$

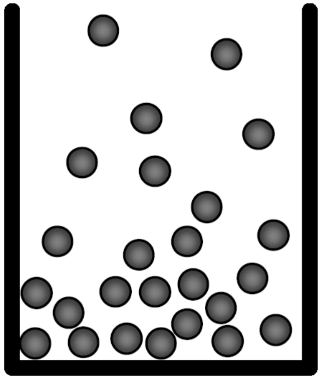
Problem #1: sedimentation & diffusion



Drag force: $F_{drag} = 6\pi\eta a v$

Gravitational force: $F_{grav} = m_{buoy} g = \left(\tfrac{4}{3}\pi a^3\right)(\Delta\rho) g$

Diffusion: $\langle \Delta r^2 \rangle = 6D\Delta t$ $\quad D = \dfrac{k_B T}{6\pi\eta a}$



1. From balance of forces, find formula for v_{sed}
2. From $m_{buoy}gh = k_BT$, find formula for scale height h
3. Find formula for time to diffuse distance a^2

Answers #1: sedimentation & diffusion

1. From balance of forces, find formula for v_{sed}

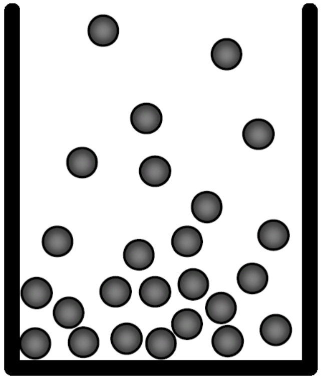
$$v_{sed} = \frac{2}{9}\frac{\Delta\rho a^2 g}{\eta} \sim a^2$$

2. From $m_{buoy} g h = k_B T$, find formula for scale height h

$$h = \frac{3}{4\pi}\frac{k_B T}{\Delta\rho a^3 g} \sim a^{-3}$$

3. Find formula for time to diffuse distance a^2

$$\tau_D = \frac{a^2}{6D} = \frac{\pi\eta a^3}{k_B T} \sim a^3$$



Meaning of Answers #1: sedimentation & diffusion

sedimentation velocity:

$$v_{sed} \sim \Delta\rho a^2 g$$

small particles sediment slowly; use centrifuge to ↑ g

scale height:

$$h \sim \frac{1}{\Delta\rho a^3}$$

strong size dependence of gravity, large particles "bad"

diffusion time:

$$\tau_D \sim a^3$$

small particles move fast

polystyrene particles in water:
$\Delta\rho$ ~ 0.05 g/cm^3, a ~ 1 µm, η ~ 10^{-3} Pa·s, kT=4·10^{-21} J

poly-methyl-methacrylate particles in density-matched solvent:
$\Delta\rho$ ~ 0.0005 g/cm^3, a ~ 1 µm, η ~ 2·10^{-3} Pa·s

sedimentation velocity: $v_{sed} = \frac{2}{9}\frac{\Delta\rho a^2 g}{\eta}$	$\approx 0.1\ \mu$m/s	$\approx$ 1 nm/s
scale height: $h = \frac{3}{4\pi}\frac{k_B T}{\Delta\rho a^3 g}$	$\approx 2\ \mu$m	$\approx 200\ \mu$m
diffusion time: $\tau_D = \frac{\pi\eta a^3}{k_B T}$	$\approx$ 0.8 s	$\approx$ 1.6 s

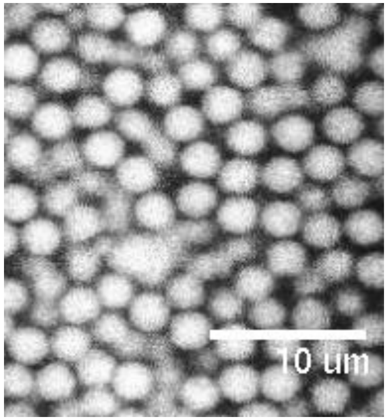
polystyrene particles in water:
$\Delta\rho$ ~ 0.05 g/cm^3, a ~ 1 µm, η ~ 10^{-3} Pa·s, kT=4·10^{-21} J

large polystyrene particles in water
$\Delta\rho$ ~ 0.05 g/cm^3, a ~ 100 µm, η ~ 10^{-3} Pa·s

sedimentation velocity: $v_{sed} = \frac{2}{9}\frac{\Delta\rho a^2 g}{\eta}$	$\approx 0.1\ \mu$m/s	$\approx$ 1 mm/s
scale height: $h = \frac{3}{4\pi}\frac{k_B T}{\Delta\rho a^3 g}$	$\approx 2\ \mu$m	$\approx$ 2 pm
diffusion time: $\tau_D = \frac{\pi\eta a^3}{k_B T}$	$\approx$ 0.8 s	$\approx$ 9 days

Important points about colloids:

- Small size important
- Understanding scaling with *a* straightforward, useful
- Granular particles (*a* > 10 μm) aren't thermal

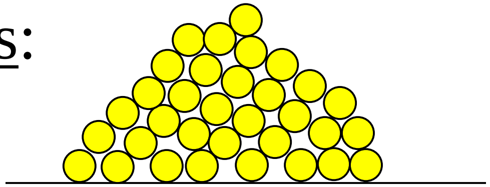


Example B: Granular materials

Definition: large, solid particles in air or vacuum

Solid-like: pile of sand
Liquid-like: pouring sand from bucket
Gas-like: throw sand into the air

Key differences from colloids:



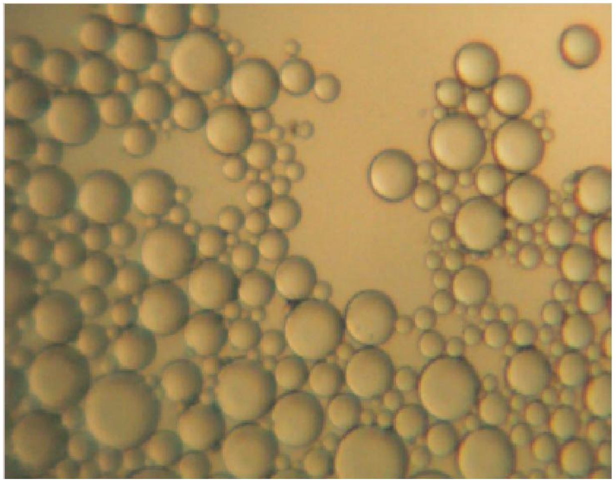
- Friction important
- $k_B T$ not important
- Gravity important (unless 2D horizontal system, or microgravity, or simulation)

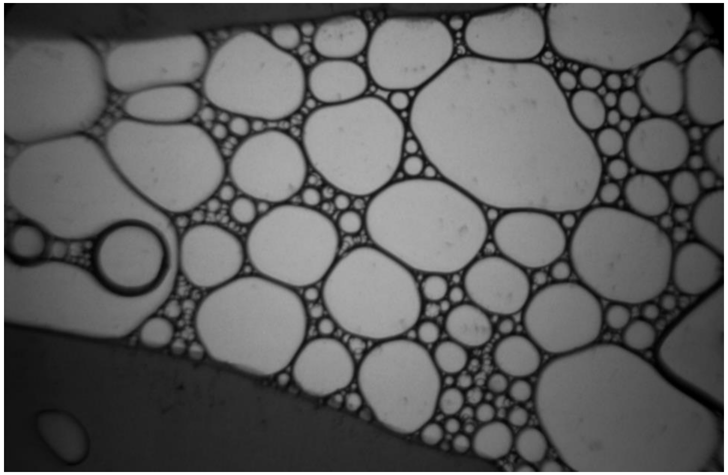
www.corbis.com

Example C: Emulsions

Definition: Liquid droplets in another liquid (oil & water)
Add surfactant (= soap) to prevent coalescence of droplets

Examples: mayonnaise (surfactant = egg), butter

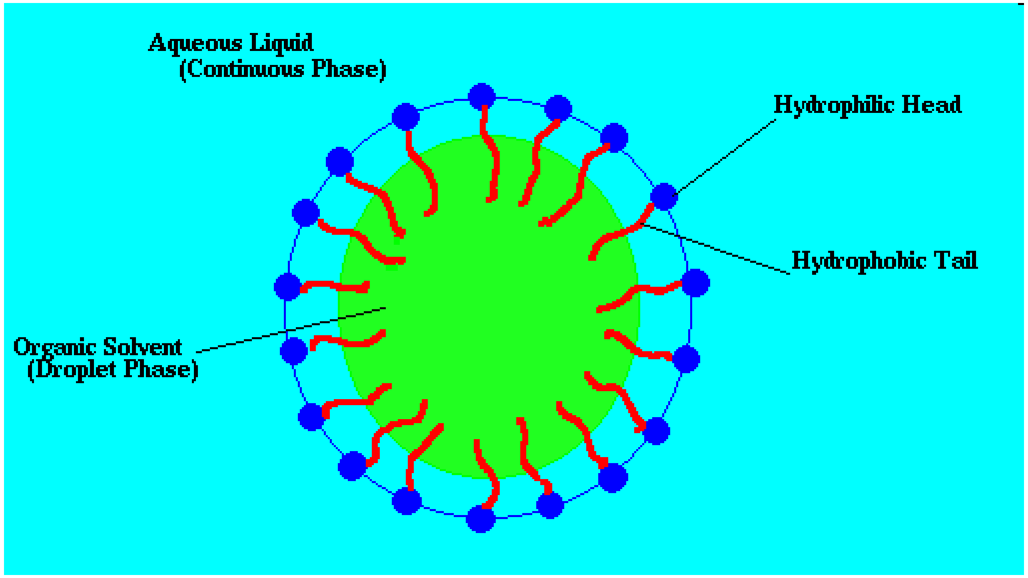




Example C: Emulsions

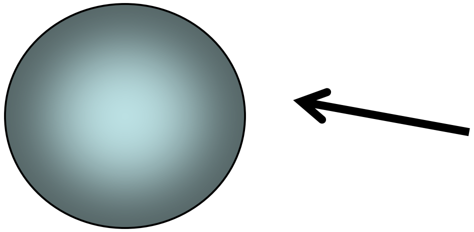
Liquid droplets in another liquid (oil & water)
Add surfactant to prevent coalescence of droplets

Key differences from colloids: droplets can deform; surfactants control surface tension which controls deformability



What is surface tension?

Surface tension γ = energy cost per unit area



Energy of surface ~ γa^2

To make emulsion with droplets of radius a:

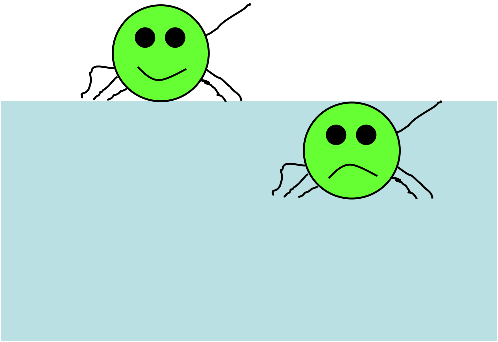
total volume = V, number of droplets $N \sim Va^{-3}$, total area $A \sim Va^{-1}$

→ thus requires energy E ~ $V\gamma/a$ to make emulsion

Question #2:

Why can bugs walk on water and I can't?

Surface tension γ = energy cost per unit area



Approximate bug as sphere of radius a.

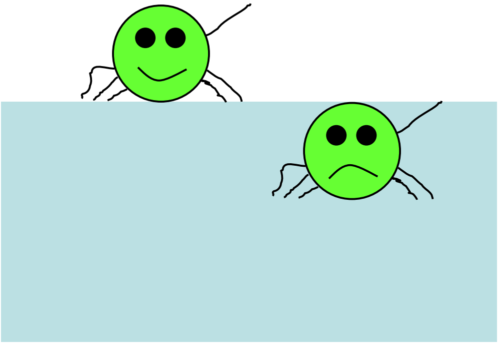
Immersing bug releases gravitational potential energy ΔGPE. This is good. Bugs should fall down.

Putting bug in water costs surface energy ΔSE. This is bad, surface energy wants to keep bug dry.

ΔGPE > ΔSE: bug gets wet!

Answer #2:

Why can bugs walk on water and I can't?



How to use ΔGPE and ΔSE to answer top question?

$$\Delta GPE \sim a^4 \qquad \Delta SE \sim a^2$$

thus, $\Delta GPE \,/\, \Delta SE \sim a^2$

if a small enough, SE penalty more significant than GPE reward

$\Delta GPE = \Delta SE$ for a ~ 4 mm: largest spherical bug that can walk on water

Laplace Pressure of Droplets

Consider balloon: balloon compresses until internal pressure sufficiently high

$$\Delta P = \frac{\gamma}{a}$$

Thus, smaller emulsion droplets are at higher pressure.

image from Wikipedia

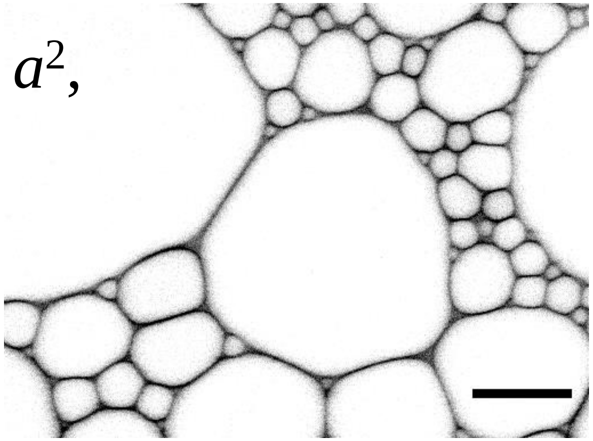
Implications for emulsion droplets:

- Droplets like to be round (minimize surface energy)

- Very large droplets sag under their own weight

- Smaller droplets are stiffer

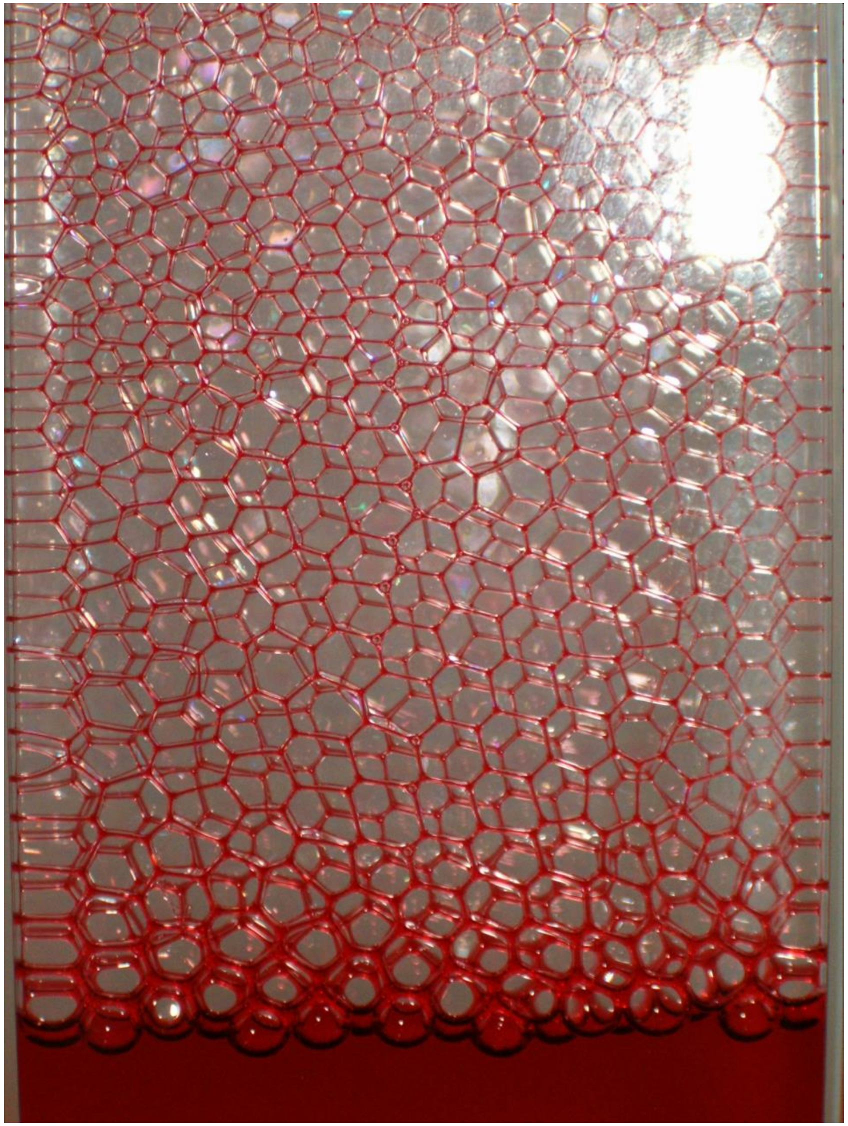
- Surfactants modify γ, but energy scales as a^2, thus size is more influential



10 μm

picture: C Hollinger & E Weeks

Example D: Foams



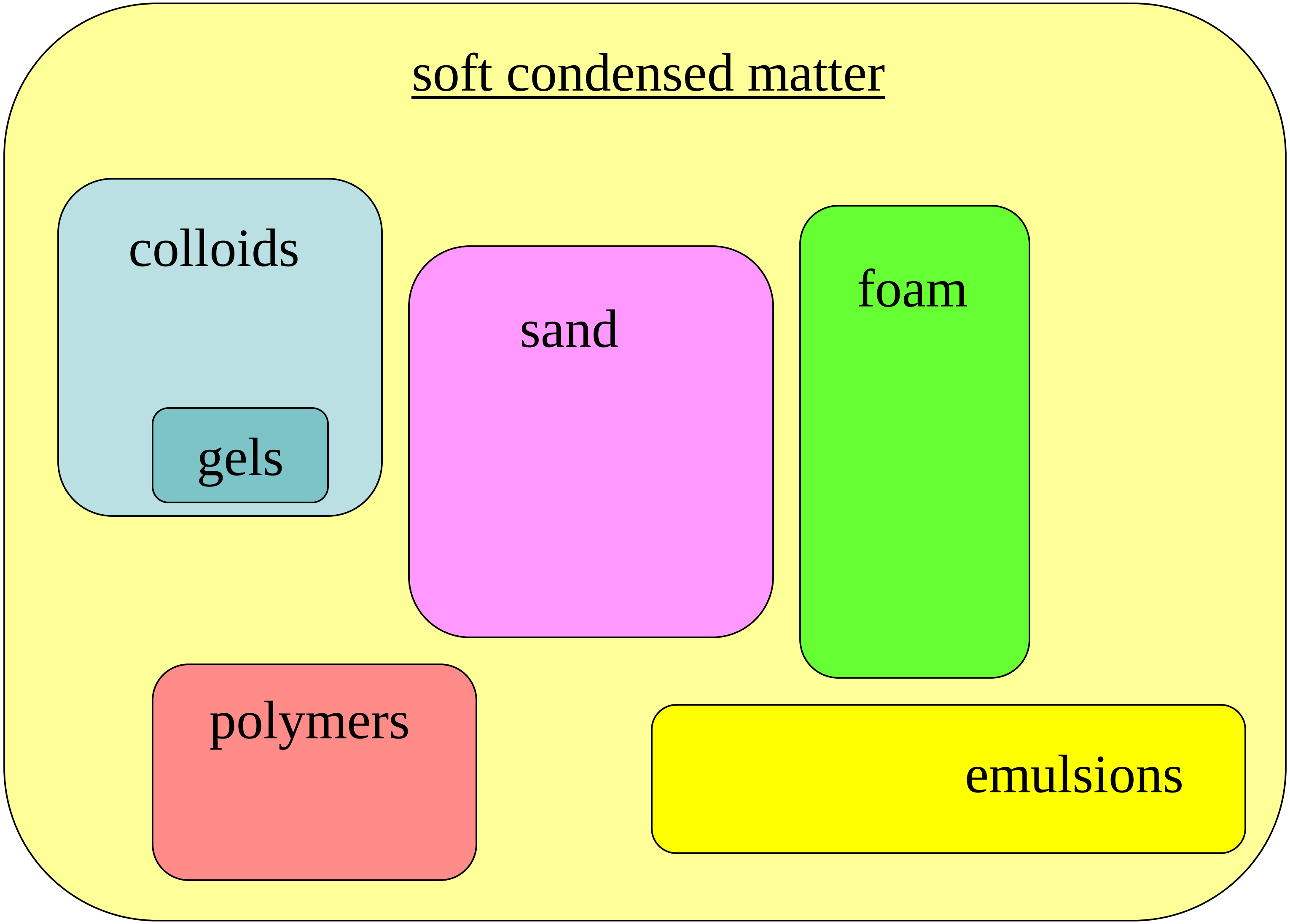
Definition: Like emulsions, but gas bubbles rather than droplets. Still need surfactant.

Troubles:

- Foams “coarsen” as gas diffusions from small bubbles to large (due to Laplace pressure)

- Foams drain (liquid is heavy)

Picture: G Cianci & E Weeks



soft condensed matter

colloids

gels

sand

foam

polymers

emulsions

Conclusion:
Life is full of interesting materials