PHYSICAL FLUID DYNAMICS

Second Edition

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INTRODUCTION

1.1 Preamble

We know from everyday observation that liquids and gases in motion behave in very varied and often complicated ways. When one observes them in the controlled conditions of a laboratory, one finds that the variety and complexity of flow patterns arise even if the arrangement is quite simple. Fluid dynamics is the study of these phenomena. Its aims are to know what will happen in a given arrangement and to understand why.

One believes that the basic physical laws governing the behaviour of fluids in motion are the usual well-known laws of mechanics—essentially conservation of mass and Newton's laws of motion, together with, for some problems, the laws of thermodynamics. The complexities arise in the consequences of these laws. The way in which observed flow patterns do derive from the governing laws is often by no means apparent. A large theoretical and conceptual structure, built, on the one hand, on the basic laws and, on the other hand, on experimental observation, is needed to make the connection.

In most investigations in fluid mechanics, the physical properties of the fluid, its density, viscosity, compressibility, etc., are supposed known. The study of such properties and the explanation in terms of molecular structure of their values for different fluids do, of course, constitute another important branch of physics. However, the overlap between the two is, in most cases, slight. This is essentially because most flows can be described and understood entirely in macroscopic terms without reference to the molecular structure of the fluid.

Thus, for the most part, fluid dynamical problems are concerned with the behaviour, subject to known laws, of a fluid of specified properties in a specified configuration. One might for example want to know what happens when oil flows through a pipe as the result of a pressure difference between its ends; or when a column of hot air rises above a heat source; or when a solid sphere is moved through a tank of water—the whole of which might be placed on a rotating table. Ideally one would like to be able to solve such problems through an appropriate mathematical formulation of the governing laws; the role of experiment would then be to check that solutions so obtained do correspond to reality. In fact the mathematical difficulties are such that a formal theory often has to be supplemented or replaced by experimental observations and measurements. Even in cases where a fairly full mathematical description of a flow has been obtained, this has often been possible only after experiments have indicated the type of theory needed. The subject involves an interplay between theory and experiment. The proportion each contributes to our understanding of flow behaviour varies greatly from topic to topic. This book is biased towards some of those topics where experimental work has been particularly important.

On the other hand, the book is primarily concerned with 'pure' fluid mechanics rather than with applications. It attempts to develop an understanding of the phenomena of fluid flow by considering simple configurations—simple, that is, in their imposed conditions—rather than more complicated ones that might be of importance in particular applications. This is not to deny the influence of applications on the development of the subject. Fluid dynamics has numerous and important applications to engineering, to geo- and astrophysics, and to biophysics; if this is not evident a glance forward to Chapter 26 will make the point. The topics chosen for investigation, even in fundamental studies, owe much to the applications currently considered significant. For example, it is doubtful whether the intriguing phenomena that arise when the whole body of the fluid is rotating would have received much attention but for the importance of such effects in atmospheric and oceanic motions.

1.2 Scope of book

Fluid dynamics has many facets. It is necessary in any book to make some restrictions to the range of topics considered. The principal restrictions in this book are the following six.

1. The laws of classical mechanics apply throughout. Since other restrictions will limit the flows considered to low speeds, the exclusion of relativistic effects is not significant. The exclusion of quantum mechanical effects just means that we are not dealing with liquid helium.

2. The length scale of the flow is always taken to be large compared with the molecular mean-free-path, so that the fluid can be treated as a continuum. More precise meaning will be given to this statement in Section 5.2. It excludes the flow of gases at very low pressures (rarefied gases) from our considerations.

3. We consider only incompressible flow; that is flow in which the pressure variations do not produce any significant density variations. In isothermal flows this means that the density is a constant; in other flows that it is a function of the temperature alone.

There are two ways in which fulfilment of this condition can come about. The fluid may have such a small compressibility (such a large bulk modulus) that, even if large pressure variations are present, they produce only slight density variations. Or the pressure variations may be sufficiently small that, even if the compressibility is not so small, the density variations are small. Liquid flows can usually be treated as incompressible for the former reason. More surprisingly, gas flows can often be similarly treated for the latter reason—whenever, as we shall see in Section 5.8, the flow speed is everywhere low compared with the speed of sound. Thus, although this restriction excludes the many interesting phenomena of the high-speed flow of gases (compressible flow), it still retains a wide range of important situations in the dynamics of gases as well as of liquids.

4. We consider only Newtonian fluids. This is a statement about the physical properties that affect the stresses developed within a fluid as a result of its motion and thus enter the dynamics of a flow. To see what is meant by a Newtonian fluid we consider a simple configuration, Fig. 1.1; the relationship of this to a general flow configuration, and thus a more rigorous definition of a Newtonian fluid, will be considered in Section 5.6. In Fig. 1.1, all the fluid is moving in the same direction but with a speed that varies in a perpendicular direction; i.e. the only non-zero component of the velocity is the x-component, u, and this is a function, $u(y)$, of the coordinate y. Across any arbitrary plane, perpendicular to y, within the fluid, a stress will act. As drawn in Fig. 1.1, the faster fluid above the plane AB will drag the fluid below forward, and the slower fluid below will drag the fluid above back. Equal and opposite forces will thus act on the fluid above and below as shown (although the arrows are drawn on the sides of the fluid *on* which they act, the lines of action of

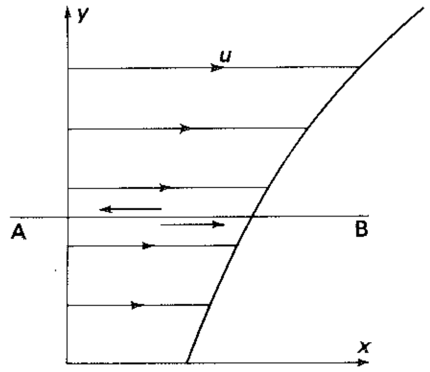

Fig. 1.1 Schematic diagram of viscous stress generated by simple velocity variation, $u(y)$. Short arrows represent forces acting in plane of AB, drawn on side of fluid on which force acts.

both forces are actually in the dividing plane AB). The generation of this internal stress is known as viscous action. In a Newtonian fluid the stress is directly proportional to the velocity gradient; if τ is the force per unit area,

$$\tau = \mu \frac{\partial u}{\partial y} \tag{1.1}$$

where μ is the coefficient of viscosity of the fluid, often called just the viscosity. Another way of stating the definition of a Newtonian fluid is to say that it is a fluid of constant viscosity. Constant here means that it does not depend on the velocity field; the viscosity of a Newtonian fluid may (and usually does) vary with temperature or it may (but usually does not) vary with pressure.

All gases and liquids with small molecules interacting in a simple way are observed to be very closely Newtonian. Non-Newtonian behaviour can arise in liquids with long molecules (polymers), in solutions of polymers, in liquids where the molecules tend to gather in more organized structures than usual, and in suspensions and emulsions (the former name being used when solid material is dispersed through a liquid and the latter when drops of one liquid are dispersed through another). Thus all the common fluids one is likely to use in the laboratory are Newtonian, as are many of the fluids occurring in important applications. However, by restricting ourselves to Newtonian fluids, we are excluding various fluids of importance in biophysics (e.g. blood flowing through small capillaries), industrial applications (e.g. many paints), and chemical engineering.

5. We shall not (with two exceptions, Sections 4.5 and 17.2) consider flows with free surfaces. Topics such as waves on a water surface, flow in open channels, and the dynamics of bubbles and drops are thus outside the scope of the book.

6. We shall not consider any problems in which electromagnetic effects are important, either purely electrohydrodynamic problems (such as the behaviour of a liquid of variable dielectric constant in an electric field) or magnetohydrodynamic problems (such as the flow of an electrically conducting fluid in a magnetic field).

All the above restrictions mean, in varying degrees, that important and interesting topics are being omitted. But the range of phenomena that remains within our scope is wide and varied. We shall be dealing primarily with flows induced by imposed pressure gradients, flows arising from the relative motion of boundaries or of one boundary and ambient fluid, convection—that is, flows induced by or associated with temperature variations—and flows strongly affected by rotation of the whole system or by density stratification. This may not at first sight seem a very broad spectrum of topics, but each has many facets. A major reason for this is the frequent occurrence of instabilities, leading to the breakdown of one type of flow into another. The next three chapters will illustrate the variety and complexity of the phenomena that occur, and illustrate in particular that the complexity can arise even for very simple imposed conditions. We may also remark here that the examples of applications of fluid dynamics in Chapter 26, drawn from widely assorted branches of applied science, have all been chosen as cases that can be understood, at least in part, within the limitations of this book.

Throughout most of this book, except where specific experimental arrangements are concerned, we shall talk about fluids, without making any distinction between liquids and gases. This is because the range of situations within the above limitations is just the range in which liquids and gases exhibit the same phenomena (provided the comparison is made in the correct quantitative way—see Chapter 7); circumstances in which any one of the limitations, except (6), did not apply would normally refer specifically either to a liquid or to a gas. Points (3) and (4) are particularly important. The facts that in many flows gases behave as if they were incompressible and that the same law of viscous behaviour applies to gases and many liquids are central to the development of a common dynamical description for the two phases.

1.3 Notation and definitions

A list of symbols used in this book is given on pages 462–8. This is intended as an aid when a symbol reappears in the text some time after it has been first introduced.

Here we may just note the symbols for the basic fluid dynamical quantities appearing throughout the book. Fluid velocity is denoted by $\boldsymbol{u}$ (with Cartesian components (u, v, w) in directions (x, y, z)), the pressure by p, the temperature by T, fluid density by ρ, the viscosity (already introduced in Section 1.2) by μ, the kinematic viscosity (μ/ρ) by ν, and time by t.

When Cartesian coordinates are used, their choice is made on the following basis. If there is one predominant flow direction, this is the x-direction. For two-dimensional flow (see below) the coordinates are x and y. When gravitational forces are significant, the z-direction is vertically upwards; this choice overrides the two preceding sentences when they would imply a different choice.

Many of the dynamical and physical quantities appearing in fluid dynamics are standard quantities with accepted definitions familiar to a physicist. It has not been thought necessary to define such quantities in the text, but their dimensions are quoted in the list of symbols when this provides a useful reminder of the definition.

It is conventional to use a double letter symbol for most non-dimensional quantities (e.g. Re for Reynolds number, Ra for Rayleigh number). Such symbols will be printed non-italicized to distinguish them from products of two quantities.

Two terms that will be used frequently require definition. Both refer to particular classes of flow that are often considered because of their relative simplicity.

Firstly, a steady flow is one which does not change with time. An observer looking at such a flow at two different instants will see exactly the same flow pattern, although the fluid at each position in this pattern will be different at the two instants. Mathematically (using the Eulerian system to be introduced in Section 5.3), steady flow may be expressed

$$\partial/\partial t = 0 \tag{1.2}$$

where the derivative operates on any parameter associated with the flow. Steady flow can occur only if all the imposed conditions are constant in time. This means that a flow is steady only in the appropriate frame of reference (flow past a fixed obstacle may be steady, but the same situation seen as the obstacle moving through the fluid is not steady, even though the two cases are dynamically equivalent—see Section 3.1). However, one would normally choose that frame for study of the flow. A flow that is changing with time is, of course, called unsteady. An intrinsically unsteady flow is one that is not steady in any frame of reference. Such a flow must occur if there is no frame in which the imposed conditions remain fixed. We shall be seeing that intrinsically unsteady flow also sometimes arises spontaneously even when the imposed conditions are steady.

Secondly, a two-dimensional flow is one in which the motion is confined to parallel planes (the velocity component in the perpendicular direction is zero everywhere) and the flow pattern in every such plane is the same. Formally,

$$w = 0, \quad \partial/\partial z = 0. \tag{1.3}$$

Such a motion may occur in an effectively two-dimensional geometry with the ends in the third direction so distant that they have negligible effect on the flow in the region of interest.

The significance of these concepts will become clearer through specific examples in the following chapters.

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PIPE AND CHANNEL FLOW

2.1 Introduction

In this and the next two chapters, we take three geometrically simple flow configurations and have a look at the principal flow phenomena. These will provide a more specific introduction than the last chapter to the character of fluid dynamics. We consider these examples now, before starting on the formal development of the subject in Chapter 5; we can then approach the setting up of the equations of motion with an idea of the types of phenomena that one hopes to understand through these equations. Although these chapters are primarily phenomenological, the present chapter will also be used to introduce some simple theoretical ideas.

The first topic is viscous incompressible flow through pipes and channels. Consider a long straight pipe or tube of uniform circular cross-section. One end of this is supplied by a reservoir of fluid maintained at a constant pressure, higher than the constant pressure at the other end. A simple arrangement for doing this in principle, using a liquid as the working fluid, is shown in Fig. 2.1; practical arrangements for investigating the phenomena to be described require some refinement of this arrangement. Fluid is pushed through the pipe from the high pressure end to the low. We suppose that the gravitational force on the fluid is irrelevant, either because the pipe is horizontal or because this force is small compared with the forces associated with the pressure differences. Although there are other experiments that one can do with pipes, this configuration is usually known as pipe flow.

Channel flow is the two-dimensional counterpart of pipe flow. Flow is supposed to occur between two parallel planes close together. The pressure difference is maintained between two opposite sides of the gap. The other two sides must be walled but are supposed to be so far away from the working region that they have no effect there. This is obviously a more difficult arrangement to set up experimentally, and the description of observations will be given in the context of pipe flow. However, a simple piece of theory can be developed about one possible flow pattern, and it is convenient to consider this first for channel flow.