It is conventional to use a double letter symbol for most non-dimensional quantities (e.g. Re for Reynolds number, Ra for Rayleigh number). Such symbols will be printed non-italicized to distinguish them from products of two quantities.

Two terms that will be used frequently require definition. Both refer to particular classes of flow that are often considered because of their relative simplicity.

Firstly, a steady flow is one which does not change with time. An observer looking at such a flow at two different instants will see exactly the same flow pattern, although the fluid at each position in this pattern will be different at the two instants. Mathematically (using the Eulerian system to be introduced in Section 5.3), steady flow may be expressed

$$\partial/\partial t = 0 \qquad (1.2)$$

where the derivative operates on any parameter associated with the flow. Steady flow can occur only if all the imposed conditions are constant in time. This means that a flow is steady only in the appropriate frame of reference (flow past a fixed obstacle may be steady, but the same situation seen as the obstacle moving through the fluid is not steady, even though the two cases are dynamically equivalent—see Section 3.1). However, one would normally choose that frame for study of the flow. A flow that is changing with time is, of course, called unsteady. An intrinsically unsteady flow is one that is not steady in any frame of reference. Such a flow must occur if there is no frame in which the imposed conditions remain fixed. We shall be seeing that intrinsically unsteady flow also sometimes arises spontaneously even when the imposed conditions are steady.

Secondly, a two-dimensional flow is one in which the motion is confined to parallel planes (the velocity component in the perpendicular direction is zero everywhere) and the flow pattern in every such plane is the same. Formally,

$$w = 0, \quad \partial/\partial z = 0. \qquad (1.3)$$

Such a motion may occur in an effectively two-dimensional geometry with the ends in the third direction so distant that they have negligible effect on the flow in the region of interest.

The significance of these concepts will become clearer through specific examples in the following chapters.

2

PIPE AND CHANNEL FLOW

2.1 Introduction

In this and the next two chapters, we take three geometrically simple flow configurations and have a look at the principal flow phenomena. These will provide a more specific introduction than the last chapter to the character of fluid dynamics. We consider these examples now, before starting on the formal development of the subject in Chapter 5; we can then approach the setting up of the equations of motion with an idea of the types of phenomena that one hopes to understand through these equations. Although these chapters are primarily phenomenological, the present chapter will also be used to introduce some simple theoretical ideas.

The first topic is viscous incompressible flow through pipes and channels. Consider a long straight pipe or tube of uniform circular cross-section. One end of this is supplied by a reservoir of fluid maintained at a constant pressure, higher than the constant pressure at the other end. A simple arrangement for doing this in principle, using a liquid as the working fluid, is shown in Fig. 2.1; practical arrangements for investigating the phenomena to be described require some refinement of this arrangement. Fluid is pushed through the pipe from the high pressure end to the low. We suppose that the gravitational force on the fluid is irrelevant, either because the pipe is horizontal or because this force is small compared with the forces associated with the pressure differences. Although there are other experiments that one can do with pipes, this configuration is usually known as pipe flow.

Channel flow is the two-dimensional counterpart of pipe flow. Flow is supposed to occur between two parallel planes close together. The pressure difference is maintained between two opposite sides of the gap. The other two sides must be walled but are supposed to be so far away from the working region that they have no effect there. This is obviously a more difficult arrangement to set up experimentally, and the description of observations will be given in the context of pipe flow. However, a simple piece of theory can be developed about one possible flow pattern, and it is convenient to consider this first for channel flow.

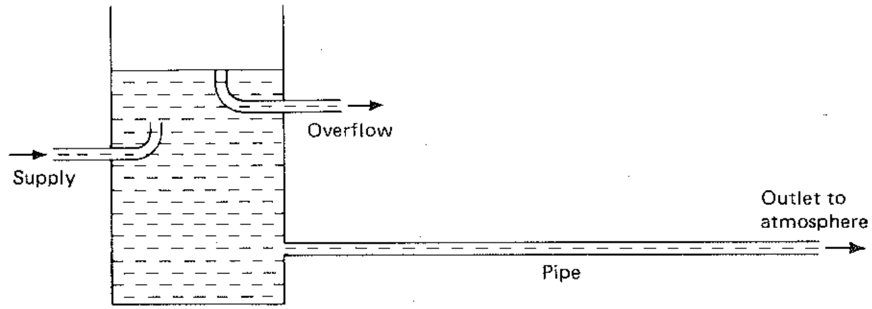

FIG. 2.1 Simple pipe flow: experimental arrangement. The pipe length is reduced in scale.

2.2 Laminar flow theory: channel

Figure 2.2 shows the notation for this theory. The channel width is taken as $2a$ and its length as l; we are supposing that l is much larger compared with a than one can show in the diagram. Pressures p_1 and p_2 ($p_1 > p_2$) are maintained at the ends of the channel. Coordinates are chosen as shown with the zero of y on the mid-plane of the channel.

It is observed (see Section 5.7) that the fluid immediately next to the walls remains at rest—a fact known as the no-slip condition. The speed, u, with which the fluid moves in the x-direction must thus be a function of y—zero at $y = \pm a$, non-zero elsewhere. This distribution $u(y)$ is known as the velocity profile.

The remoteness of the other walls, in the z-direction, is taken to imply that the flow is two-dimensional (see Section 1.3). We can thus consider all processes to be occurring in the plane of Fig. 2.2.

For the present theory we also suppose that the velocity profile is the same at all distances down the channel; that is at all x. Since the density ρ is taken to be constant as discussed in Section 1.2(3), this is obviously one way of satisfying the requirement that the flow should conserve mass. The same velocity profile transports the same amount of mass per unit time past every station.

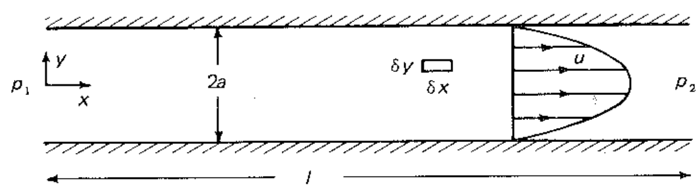

FIG. 2.2 Definition diagram for channel flow. Width of channel is shown exaggerated with respect to length.

The pressure varies with x, obviously, but is constant across the pipe at each x. We shall see below that a pressure gradient in a given direction generates a force in that direction; there is nothing to balance such a force in the y- or z-direction.

We consider now the forces acting on a small element of fluid of sides δx and δy as shown in Fig. 2.2 and side δz in the third direction. There are two processes giving rise to such forces—the action of viscosity as described in Section 1.2(4), and the pressure.

The expression for the viscous force illustrates an important general point that, although the viscous stress depends on the first spatial derivative of the velocity, the viscous force on a fluid element depends on the second derivative. The net force on the element is the small difference of the viscous stresses on either side of it. Figure 2.3 extends Fig. 1.1 to show this. Per unit area perpendicular to the y-direction, forces $\mu(\partial u/\partial y)_{y+\delta y}$ and $-\mu(\partial u/\partial y)_y$ act in the x-direction on the region between planes AB and CD. The net force on our element is

$$\left[\mu\left(\frac{\partial u}{\partial y}\right)_{y+\delta y} - \mu\left(\frac{\partial u}{\partial y}\right)_y\right]\delta x\,\delta z = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\delta y\,\delta x\,\delta z \quad \text{(when } \delta y \text{ is small enough)}$$

$$= \mu\frac{\partial^2 u}{\partial y^2}\delta x\,\delta y\,\delta z \text{ (when } \mu \text{ is constant).} \quad (2.1)$$

The viscous force per unit volume is $\mu\,\partial^2 u/\partial y^2$. In the present case, for which u is independent of x and z, this may be written $\mu\,d^2u/dy^2$. From the general shape of the velocity profile in Fig. 2.2, or from the physical expectation that viscous action will oppose the flow, we anticipate that d^2u/dy^2 will be negative.

The pressure decreases as one goes downstream; there will be slightly different pressure forces acting on the two ends of the element. Since

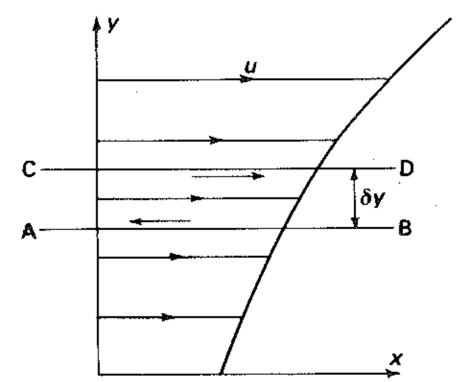

FIG. 2.3 Extension of Fig. 1.1 to show viscous stresses acting on fluid element.

pressure is force per unit area, that at the upstream end acting in the downstream direction on the element is $p_x\,\delta y\,\delta z$ (p_x denoting the value of p at x) and that at the downstream end acting in the upstream direction is

$$p_{x+\delta x}\,\delta y\,\delta z = \left(p_x + \frac{\partial p}{\partial x}\,\delta x\right)\delta y\,\delta z \tag{2.2}$$

(for small enough δx). The net force in the downstream direction is

$$-\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z \tag{2.3}$$

or $-\partial p/\partial x$ per unit volume. This will be positive.

Again the partial derivative (which has been written because we shall later be looking at this matter in a more general context) may be replaced by a total derivative, because p varies only with x. Further, the assumption of an unchanging velocity profile makes the dynamical processes the same at all stations downstream; the pressure force per unit volume—i.e. the pressure gradient—must be independent of x. Hence,

$$-\frac{\partial p}{\partial x} = -\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{p_1 - p_2}{l} = G, \text{ say.} \tag{2.4}$$

The momentum of the element $\delta x\,\delta y\,\delta z$ is not changing; each fluid particle travels downstream at a constant distance from the centre of the channel and so with a constant speed. Hence, the total force acting must be zero:

$$\mu\frac{\partial^2 u}{\partial y^2}\,\delta x\,\delta y\,\delta z - \frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z = 0, \tag{2.5}$$

that is

$$\mu\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = -G. \tag{2.6}$$

With the boundary conditions

$$u = 0 \quad \text{at} \quad y = \pm a \tag{2.7}$$

this integrates to give

$$u = \frac{G}{2\mu}(a^2 - y^2). \tag{2.8}$$

We have ascertained that the velocity profile is a parabola.

The mass of fluid passing through the channel per unit time and per unit length in the z-direction is

$$\int_{-a}^{a} \rho u\,\mathrm{d}y = 2G\rho a^3/3\mu. \tag{2.9}$$

2.3 Laminar flow theory: pipe

The corresponding flow in a pipe is usually known as Poiseuille flow (or sometimes, in the interests of historical accuracy, Hagen–Poiseuille flow). This case is marginally more complicated because of the cylindrical geometry. Figure 2.4 shows a cross-section of the pipe; the flow direction (the x-axis) is normal to the page. The velocity profile now represents the speed as a function of radius, $u(r)$, and we again consider the case when this is independent of x. We consider an element of fluid as shaded in Fig. 2.4 and having length δx in the flow direction. The viscous forces on the two faces of this now differ slightly not only because the velocity gradients differ but also because the two faces have different areas. The force on one face is

$$\mu(\partial u/\partial r) r\,\delta\phi\,\delta x \tag{2.10}$$

and, by an argument parallel to that in Section 2.2, the net viscous force on the element is

$$\mu\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\delta r\,\delta x\,\delta\phi. \tag{2.11}$$

The pressure force on one end of the element is

$$pr\,\delta\phi\,\delta r \tag{2.12}$$

and the net pressure force

$$-(\partial p/\partial x) r\,\delta x\,\delta\phi\,\delta r. \tag{2.13}$$

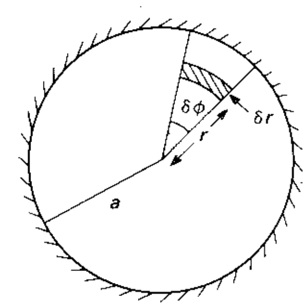

FIG. 2.4 Definition sketch for pipe flow.

Arguing in the same way as for channel flow we get

$$\mu \frac{\mathrm{d}}{\mathrm{d}r}\left(r \frac{\mathrm{d}u}{\mathrm{d}r}\right) = -Gr. \tag{2.14}$$

Integration gives

$$u = -\frac{Gr^2}{4\mu} + A \ln r + B. \tag{2.15}$$

A must be zero for the velocity not to become infinite on the axis and B can be evaluated from the fact that

$$u = 0 \quad \text{at} \quad r = a \tag{2.16}$$

giving

$$u = \frac{G}{4\mu}(a^2 - r^2). \tag{2.17}$$

The velocity profile is a paraboloid with a maximum speed

$$u_{\max} = \frac{Ga^2}{4\mu}. \tag{2.18}$$

The mass per unit time, or mass flux, passing through the pipe is

$$\int_0^a \rho 2\pi r u \, \mathrm{d}r = \frac{\pi \rho G a^4}{8\mu} = \frac{\pi \rho (p_1 - p_2) a^4}{8\mu l}. \tag{2.19}$$

This is a quantity of some importance as it can readily be measured. Agreement with observation (under circumstances to be delimited below) provides an important check of the validity of underlying hypotheses, such as the no-slip condition and the applicability of continuum mechanics (see Chapter 5). Alternatively, an unknown viscosity can be determined from the rate of flow of the fluid through a tube under a known pressure gradient. This is the principle of one important type of viscometer.

An average speed can be defined as the mass flux divided by the density and cross-sectional area

$$u_{\text{av}} = \frac{Ga^2}{8\mu}. \tag{2.20}$$

2.4 The Reynolds number

We have determined above one theoretically possible flow behaviour. It is not the only possibility. Sometimes the actual flow behaviour corresponds to the theory, sometimes not. In order to specify the circumstances in which the different types of flow occur, we need to introduce the concept of the Reynolds number.

There are several types of variable associated with the pipe flow configuration: the dimensions of the pipe, the rate of flow, the physical properties of the fluid. For present purposes, we will suppose that the situation is fully specified if we know:

$d(=2a)$, the pipe diameter
u_{av}, the average flow speed
ρ, the fluid density
μ, the viscosity.

Two omissions from this list require some comment. The pipe length is not included on the supposition that, provided the pipe is long enough, the type of flow is determined before the downstream end can have any influence. The pressure gradient is not included because it cannot be varied independently of the above parameters. To produce the same average flow-speed of the same fluid through the same pipe will require the same imposed pressure gradient—whether or not the Poiseuille flow relationship applies. Hence, the pressure gradient need not be included in the specification of the situation. It is arguable that one should include the pressure gradient and omit the average speed, since the former is the controlled variable in most experiments. In Section 18.3 we shall see that there is one case in which this would certainly be the better procedure. However, this has not been conventional practice, and to adopt it would make for confusion. We notice that u_{av} relates directly to the total rate of flow through the pipe and must be the same at every station along the length—unlike $u_{\max}$ and other speeds one might define that depend on the detailed velocity profile.

We are now concerned with the question: what type of flow occurs for given values of d, u_{av}, ρ, and μ? But these four parameters are dimensional quantities, whereas the concept of a 'type of flow' does not have dimensions associated with it. Just as it is meaningless to write down an equation, $A = B$, with A and B of different dimensions, so it is meaningless to associate a type of flow with certain values of any dimensional quantity. For example, one would not expect a particular type of flow to occur over the same range of u_{av} for pipes of different diameter or for different fluids. The values of u_{av} specifying the range must be expressible (in principle, whether or not in practice) in terms of the things that determine it. There must be some (known or unknown) expression for it, and this expression must bring in other dimensional quantities in order to be dimensionally satisfactory. Thus we look for the factors that determine the type of flow in terms of dimensionless, not dimensional, parameters. Such dimensionless parameters must be provided as combinations of the specifying dimensional parameters. (We have arrived by a plausibility argument at a conclusion that we shall be examining more systematically in Chapter 7.)

There is one dimensionless combination of the four parameters for the present configuration:

$$\mathrm{Re} = \frac{\rho u_{av} d}{\mu}. \qquad (2.21)$$

This is known as the Reynolds number of pipe flow.

It is one example of the general definition of the Reynolds number as

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{UL}{\nu} \qquad (2.22)$$

where U and L are velocity and length scales; that is, typical measures of how fast the fluid is moving and the size of the system. As the book proceeds, we shall be seeing the relevance of forms of Reynolds number to a variety of situations.

Hence, it is appropriate to discuss the different types of flow in a pipe in terms of different ranges of Reynolds number.

2.5 The entry length

When the Reynolds number is less than about 30, the Poiseuille flow theory always provides an accurate description of the flow. In fact eqn (2.19) was first established empirically by Hagen and Poiseuille and the theory was given later by Stokes—a somewhat surprising historical fact if one knows how easy it is to fail to verify the theory as a result of having the Reynolds number too high!

At higher Reynolds numbers, the Poiseuille flow theory applies only after some distance down the pipe. The fluid is unlikely to enter the pipe with the appropriate parabolic velocity profile. Consequently, there is an entry length in which the flow is tending towards the parabolic profile. At low Reynolds number, this is so short that it can be ignored. But it is found both experimentally and theoretically that, as the Reynolds number is increased, this is no longer true. The details of the entry length depend, of course, on the actual velocity profile at entry, which in turn depends on the detailed geometry of the reservoir and its connection to the pipe. However, an important case is that in which the fluid enters with a uniform speed over the whole cross-section. Because of the no-slip condition, the fluid next to the wall must immediately be slowed down. This retardation spreads inward, whilst fluid at the centre must move

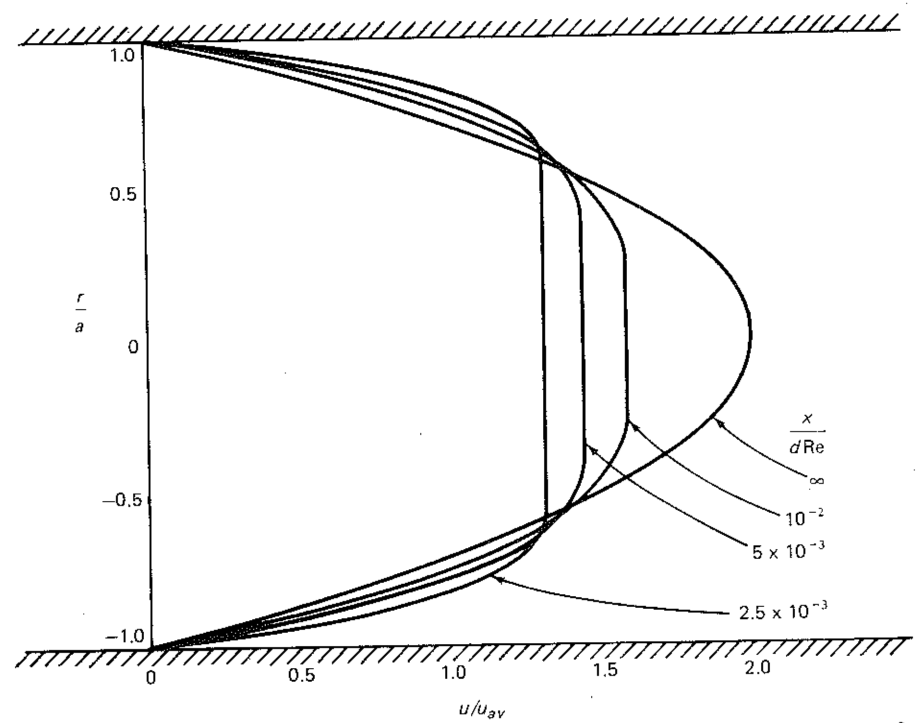

FIG. 2.5 Laminar velocity profiles in pipe entry length. Based on average of various experimental and theoretical profiles, as collected together in Ref. [347].

faster, so that the average speed remains the same and mass is conserved. One thus gets a sequence of velocity profiles as shown in Fig. 2.5. Ultimately, the parabolic profile is approached and from there onwards the Poiseuille flow theory applies.

We can use this case to illustrate the dependence of the extent of the entry length on the Reynolds number. Defining X as the distance downstream from the entry at which u_{max} is within 5 per cent of its Poiseuille value [347],

$$\frac{X}{d} \approx \frac{\mathrm{Re}}{30}. \qquad (2.23)$$

This means, for example, that for flow at a Reynolds number of 10^4 (chosen as a high value at which this type of flow can occur) in a pipe of diameter 3 cm the entry length is 10 m. Evidently there will be many practical situations in which the Poiseuille flow pattern is never reached. Even if it is reached, it will often not occupy a sufficiently large fraction of the length for eqn (2.19) to be a good approximation to the relationship between pressure drop and mass flux.

Incidentally, the length at the other end over which the presence of the outlet has an effect is always relatively small.

2.6 Transition to turbulent flow

The above is an important, but perhaps rather uninteresting, limitation to the occurrence of Poiseuille flow. As the Reynolds number is increased further, there is a much more dramatic change in the flow. It undergoes transition to the type of motion known as turbulent, the flows considered so far being called laminar.

In laminar pipe flow the speed at a fixed position is always the same. Each element of fluid travels smoothly along a simple well-defined path. (In Poiseuille flow, this is a straight line at a constant distance from the axis; in the entry length it is a smooth curve.) Each element starting at the same place (at different times) follows the same path.

When the flow becomes turbulent, none of these features is retained. The flow develops a highly random character with rapid irregular fluctuations of velocity in both space and time. An example of the way in which (one component of) the velocity at a fixed position fluctuates is shown in Fig. 2.6. An element of fluid now follows a highly irregular distorted path. Different elements starting at the same place follow different paths, since the pattern of irregularities is changing all the time. These variations of the flow in time arise spontaneously, although the imposed conditions are all held steady.

Figure 2.7 shows the effect of the changeover from laminar to turbulent flow on dye introduced continuously near the entry of a pipe in a streak thin compared with the radius of the pipe. (This arrangement is, in its essentials, the one used by Reynolds [318] in the experiments that may be regarded as the genesis of systematic studies of transition to turbulence—although there had been some earlier work by Hagen.) When the flow is laminar, the dye just travels down the pipe in a straight or almost straight line as shown in the upper picture. As the flow rate is increased, thus increasing the Reynolds number, the pattern changes as

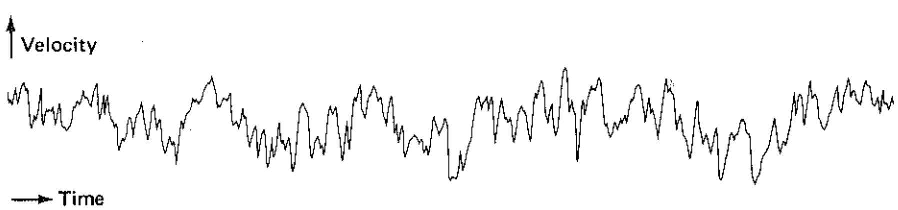

Fig. 2.6 Example of velocity variations in turbulent flow.

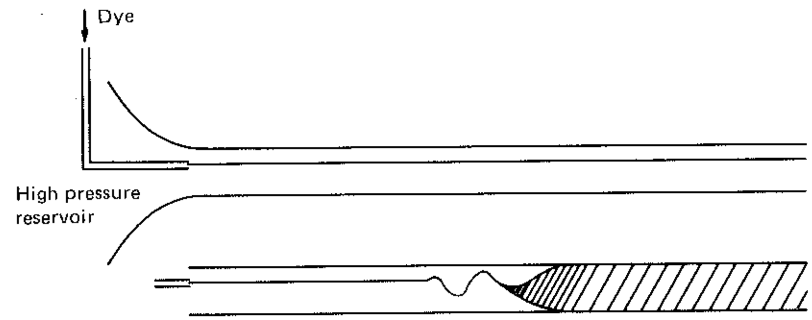

Fig. 2.7 Dye streaks in laminar and turbulent pipe flow. (Pipe length compressed relative to other dimensions.)

shown in the lower picture. The dye streak initially travels down the pipe in the same way as before, but, after some distance, it wavers and then suddenly the dye appears diffused over the whole cross-section of the pipe. The motion has become turbulent and the rapid fluctuations have mixed the dye up with the undyed fluid.

The distance fluids travels downstream before becoming turbulent varies with time, and, at any instant, there can be a laminar region downstream of a turbulent region. This comes about in the following way. The turbulence is generated initially over a small region. This region is actually localized radially (close to the wall) and azimuthally as well as axially. However, it quickly spreads over a cross-section of the pipe, and there is then a short length of turbulent flow with laminar regions both upstream and downstream of it (Fig. 2.8). This short length is known as a

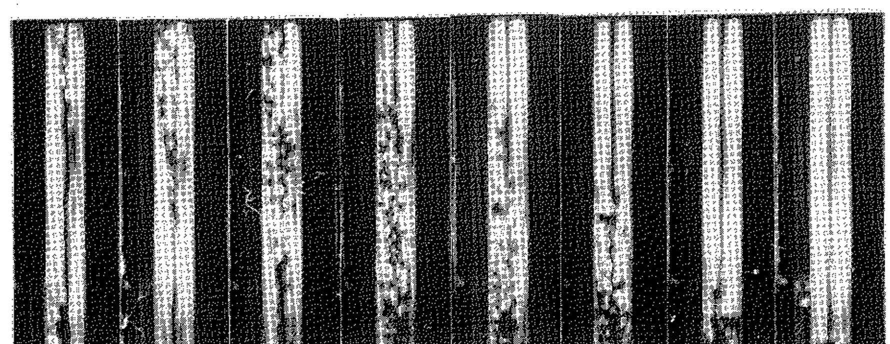

Fig. 2.8 Photo sequence showing passage of turbulent slug past fixed observation point, similar to those in Refs. [254, 256]. Flow is downwards; time increases from left to right. Slug enters field of view at top of first frame (which also shows end of earlier slug at bottom) and leaves it at bottom of seventh frame. Flow visualization by addition of small amount of birefringent material—refractive index for polarized light is affected by shear (Refs. [36, 253]).

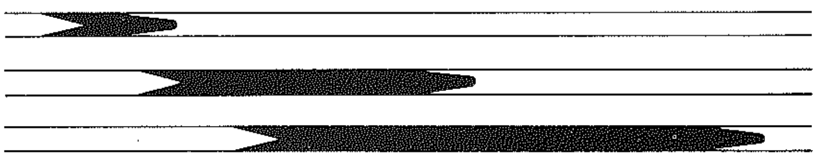

FIG. 2.9 Growth and transport of turbulent slug. Shaded regions are turbulent, unshaded laminar. The mean fluid speed is approximately midway between the speeds of the front and rear of the slug.

turbulent slug, a name that has superseded an earlier one of turbulent plug. (We postpone till Section 18.3 further consideration of the origin of turbulent slugs.) The turbulence then spreads—the laminar fluid next to each end of the turbulent slug is brought into turbulent motion—and the slug gets longer. The fluid is, of course, meanwhile travelling down the pipe. Hence, the development of a slug is as shown in Fig. 2.9; the shapes of the interfaces are indicated roughly [376, 414]. As the slug grows the interfaces soon occupy a short length of the pipe compared with the slug itself, and so laminar and turbulent regions are well demarcated.

After a while another slug is born in a similar way. By this time the previous slug has moved off downstream so there is again laminar fluid downstream of the new slug. The slugs sometimes originate randomly in time, sometimes periodically; the circumstances in which the two cases occur will be discussed further in Section 18.3. When the front interface of one slug meets the rear interface of another, as a result of their growth, the two simply merge to give a single longer slug.

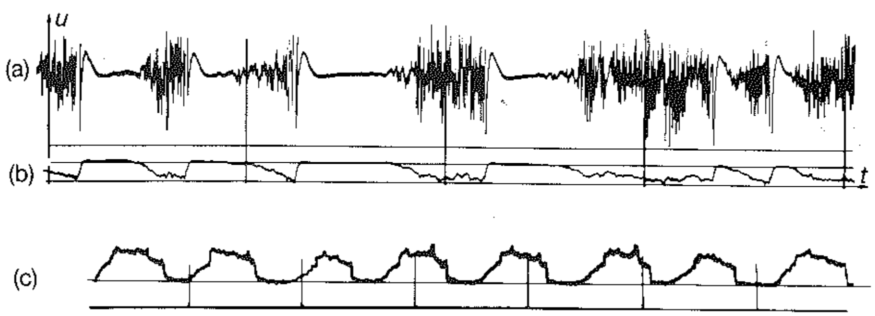

FIG. 2.10 Oscillograms of velocity fluctuations at the centre of a pipe. Traces (a) and (b) (from Ref. [327]) show random slug production for Re = 2550, $l/d = 322$; trace (a) was given by a.c. amplification of the signal and shows the velocity fluctuations in the slugs; trace (b) was given by d.c. amplification of the same signal and shows principally the local mean velocity change between laminar and turbulent flow. Trace (c) (from Ref. [296]) is the counterpart of trace (b) for periodic slug production for Re = 5000, $l/d = 290$. Note: velocity increases upwards in traces (a) and (b) but decreases upwards in trace (c).

As a result of these processes, a sensor at a fixed point in the pipe observes alternately laminar and turbulent motion. Figure 2.10 shows oscillograms of velocity fluctuations arising in this way, for random and for periodic slug production. The abrupt changes between laminar and turbulent motion illustrate again the sharpness of the interfaces.

The fraction of the time that the motion is turbulent—known as the intermittency factor—increases with distance downstream as a result of the growth of the slugs. Far enough downstream, in a long enough pipe, the laminar regions have all been absorbed and the flow is fully turbulent.

From the considerations earlier in this section, one would expect that there should be a critical value of the Reynolds number, below which the flow is wholly laminar, above which transition to turbulence occurs. In fact, the situation is more complicated than that. The transition process is extremely sensitive to the detailed geometry of the entry from the reservoir and to the level of small disturbances in the incoming fluid. As a result, transition has been observed to start at values of the Reynolds number ranging from 2×10^3 to 10^5. (This will be discussed further in Section 18.3.) The implication of this variation is not that the reasoning establishing the role of the Reynolds number was erroneous, but that dimensional parameters other than the four listed (d, u_{av}, ρ, and μ) are relevant to transition.

2.7 Relationship between flow rate and pressure gradient

The pressure difference needed to produce a given flow rate through a pipe is larger when the flow is turbulent than when it is laminar. This is shown in Fig. 2.11. Since the abscissa is the Reynolds number, it is appropriate that the ordinate should be a non-dimensional form of the average pressure gradient $(p_1 - p_2)/l$, chosen† here as $(p_1 - p_2)d^3\rho/\mu^2 l$. The dotted lines in Fig. 2.11 show this parameter plotted (logarithmically) against Reynolds number for Poiseuille flow and for wholly turbulent flow. The continuous line shows the behaviour for an actual case. At low Reynolds number it follows the Poiseuille flow line; as transition starts it rises above this, and, ultimately, when transition is

† A more conventional choice would be $(p_1 - p_2)d/\rho u_{av}^2 l$ (the above parameter divided by $(\mathrm{Re})^2$). However, because it does not contain u_{av}, our choice is more convenient for certain considerations in Section 18.3. It also shows the behaviour in a particular pipe more directly.

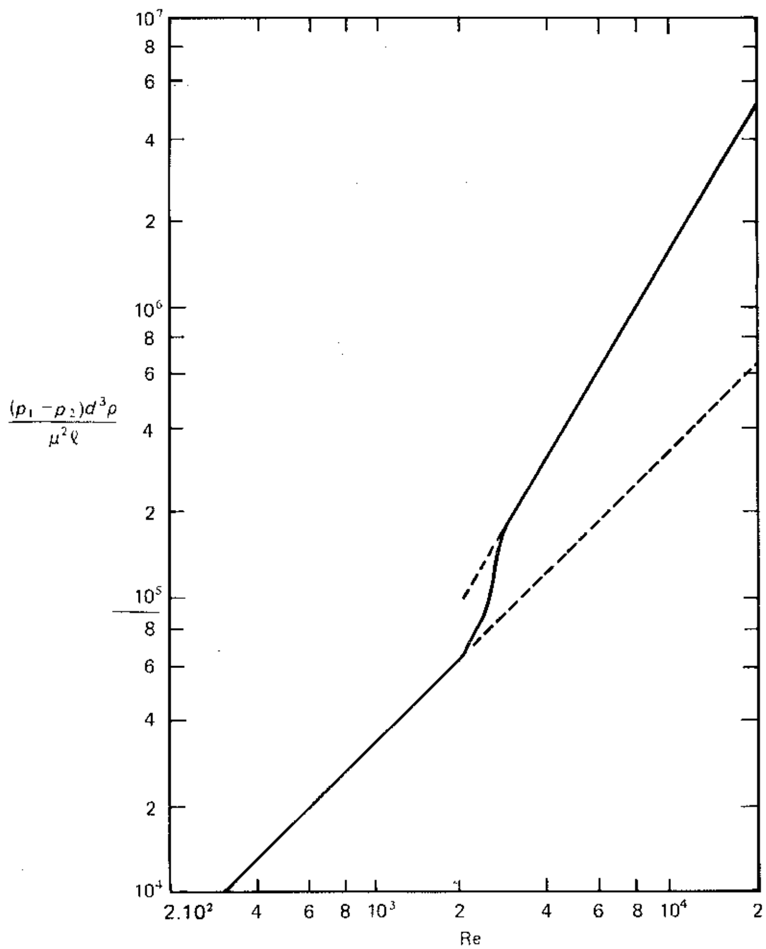

FIG. 2.11 Variation of non-dimensional average pressure gradient with Reynolds number for pipe flow. Dotted lines: wholly laminar flow and wholly turbulent flow. Full line: example of actual case. Based on information in Refs. [44, 327].

complete in a short fraction of the total length, approaches the turbulent flow line. (The case chosen shows transition at a relatively low Reynolds number. With higher Reynolds number transition in a pipe of practicable length, there is likely to be some departure from the Poiseuille law before transition because of the importance of the entry length.)

3

FLOW PAST A CIRCULAR CYLINDER

3.1 Introduction

Relative motion between some object and a fluid is a common occurrence. Obvious examples are the motion of an aeroplane and of a submarine and the wind blowing past a structure such as a tall building or a bridge. Practical situations are, however, usually geometrically complicated. Here we wish to see the complexities of the flow that can arise even without geometrical complexity. We therefore choose a very simple geometrical arrangement, and one about which there is a lot of information available.

This is the two-dimensional flow past a circular cylinder. A cylinder of diameter d is placed with its axis normal to a flow of free stream speed u_0; that means that u_0 is the speed that would exist everywhere if the cylinder were absent and that still exists far away from the cylinder. The cylinder is so long compared with d that its ends have no effect; we can then think of it as an infinite cylinder with the same behaviour occurring in every plane normal to the axis. Also, the other boundaries to the flow (e.g. the walls of a wind-tunnel in which the cylinder is placed) are so far away that they have no effect.

An entirely equivalent situation exists when a cylinder is drawn perpendicularly to its axis through a fluid otherwise at rest. The only difference between the two situations is in the frame of reference from which the flow is being observed. (Aspects of relativity theory are already present in Newtonian mechanics.) The velocity at each point in one frame is given by the vectorial addition of u_0 onto the velocity at the geometrically similar point in the other frame (Fig. 3.1). This transformation does not change the accelerations involved or the velocity gradients giving rise to viscous forces; it thus has no effect on the dynamics of the situation. (These remarks apply only when u_0 is constant; if it is changing then one does have to distinguish between the cylinder accelerating and the fluid accelerating.)

It is, however, convenient to use a particular frame of reference for describing the flow. Except where otherwise stated, the following description will use the frame of reference in which the cylinder is at rest.